Grad Logic Problem for Exam 2

The classic solution is Gödel numbering. Take the 256 character ASCII chart and assign to each character its order in the chart...so if G is the 47th number, associate G with 47. Now suppose a statement has n characters, spaces included. Write the primes in order: $2, 3, 5, 7, 11, \ldots$. If the character in the statement has G in the i^{th} position, take the i^{th} prime to the power 47. Do the same for all n characters. Now we have some (huge) number $M = p_i^{e_1} p_2^{e_2} \cdots p_n^{e_n}$. Some of the e's can be repeated, but of course they would be applied to different primes. Conversely, given such an M, the Fundamental Theorem of Arithmetic says it can be factored into primes with various exponents essentially only one way if the primes are arranged increasingly. So if G were the first symbol in the statement, the factor 2^{47} would appear in M. By uniqueness, since 2 is the first prime, the symbol with chart number 47, namely G, would have to be first in the statement. So this function which assigns character positions on the ASCII chart to the primes in order is injective. No two values of M would generate the same character assignments, so each corresponds to a unique statement. The order of the M's induces an order on the statements.