SPRING 2026 - CALCULUS 3 - TEST #2B - Solutions

1) Find the gradient of $f(x,y,z) = \sin(e^{x\cos yz})$

$f_x = \cos(yz)e^{x\cos(yz)} \cdot \cos(e^{x\cos(yz)})$
$f_y = -xz\sin(yz)e^{x\cos(yz)} \cdot \cos(e^{x\cos(yz)})$
$f_z = -xy\sin(yz)e^{x\cos(yz)} \cdot \cos(e^{x\cos(yz)})$
so the gradient $\nabla f = e^{x\cos(yz)} \cdot \cos(e^{x\cos(yz)})\left[\cos(yz)\widehat{\mathbf{i}} - xz\sin(yz)\widehat{\mathbf{j}} - xy\sin(yz)\widehat{\mathbf{k}}\right]$

2) Find the equation of the tangent plane to the surface given by $z = x^2 + y^2$ at the point $(3,3,18)$

$\nabla(x^2+y^2-z) = 2x\widehat{\mathbf{i}} + 2y\widehat{\mathbf{j}} - k$. *Evaluated at* $(3,3,18)$, *this is* $6\widehat{\mathbf{i}} + 6\widehat{\mathbf{j}} - k$. *This is a direction vector of the plane* $6x + 6y - z = d$. *Since the plane passes thru* $(3,3,18)$, *this means* $18 + 18 - 18 = d$, *or* $d = 18$. *Then the tangent plane has equation* $6x + 6y - z = 18$.

3) If $\phi(x,y) = e^x y^3 \sin xy$, find ϕ_{xy} and ϕ_{yx}

$\phi_x = e^x y^3 \sin(xy) + e^x y^4 \cos(xy)$
$\phi_{xy} = e^x[(3y^2 - xy^4)\sin(xy) + (xy^3 + 4y^3)\cos(xy)] = y^2e^x[(3 - xy^2)\sin(xy) + (xy - 4y)\cos(xy)]$.
The function and its partials up to second order are continuous, so Clairaut's Theorem applies and $\phi_{xy} = \phi_{yx}$

4) Find all relative maxima, minima, and saddle points of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

$f_x = 3x^2 - 3 = 0$ *when* $x = \pm 1$ *and* $f_y = 3y^2 - 12 = 0$ *when* $y = \pm 2$, *so the candidate critical points are* $(1,2), (-1,2), (1,-2)$, *and* $(-1,-2)$. *Calculating* $\Delta = f_{xx}f_{yy} - f_{xy} = (6x)(6y) - 0 = 36xy$. *So at* $(1,2)$ *and* $(-1,-2)$, $\Delta > 0$ *(conclusion possible). So at* $(1,2)$, $f_{xx} > 0$ *which gives a minimum, and at* $(-1,-2)$, $f_{xx} < 0$ *which gives a maximum. At both* $(-1,2)$ *and* $(1,-2)$ *we have* $\Delta < 0$, *so these points are saddle points.*

5) Suppose $\phi = f(x,y,z,t)$, $x = x(u,t)$, $y = y(v,t)$, $z = z(u,v)$, $u = u(t)$, and $v = v(t)$. Draw the dependency diagram for $\phi'(t)$