

SPRING 2026 - CALCULUS 2 - TEST 2B - Solutions

1) You have \$1000 invested at 6% per annum continuous interest starting in year 1. At the end of year 3 you withdraw half the money in the account and leave the remainder on deposit. At the beginning of year 4, you get a better rate of 7.5% per annum. How much would you have on deposit at the end of year 10?

Use the Pert formula to find future values. After three years at 6% the \$1000 grows to \$1197.22. Removing half of that leaves \$598.61. Letting that grow for seven years at 7.5% gives \$1011.93.

2) A cup of soup cooled from 90°C to 60°C in ten minutes in a room whose temperature was 20°C . How much longer will it be until the soup is at 35°C ?

Newton's Law of Cooling is $H(t) - H_S = (H_0 - H_S)e^{-kt}$, where $H(t)$ is temperature of an object at time t after it has started at temperature H_0 to cool off or heat up in an ambient temperature of H_S . We need to find k , so we plug in our data. $H(10) = 60$, $H_S = 20$, $H_0 = 90$. So $(60 - 20) = (90 - 20)e^{-10k}$. Or $40 = 70e^{-10k}$. Solving for k we have $\ln(0.5714) = -0.5596 = -k(10)$, then $k = 0.05596$. We want the time t to cool from 60 to 35 with this cooling constant. $(35 - 20) = (60 - 20)e^{-0.05596t}$. This gives $15 = 40e^{-0.05596t}$, or $\ln(0.375) = -0.981 = -0.05596t$. We find $t = 17.5$ minutes longer.

3) Evaluate: $\int_1^4 \frac{2\sqrt{x}}{\sqrt{x}} dx$

Let $u = \sqrt{x}$. Then $du = \frac{dx}{2\sqrt{x}}$. So $\int_1^4 \frac{2\sqrt{x}}{\sqrt{x}} dx = \int_1^2 2^u (2du) = \left[\frac{2 \cdot 2^u}{\ln 2} \right]_1^2 = \frac{2}{\ln 2} [2^2 - 2] = \frac{4}{\ln 2}$.

4) Plutonium-239 has a half-life of 24,360 days. If 25 [g] of plutonium is released into the atmosphere by a nuclear accident, how many years will it take for 65% of it to decay?

Convert to years: $24360 \text{ days} = 66.7 \text{ years}$. From the half-life info, $\frac{1}{2} = e^{-\lambda(66.7)}$, where λ is the (positive) decay constant. Solving the exponential equation we get $-.69315 = -66.7\lambda$, or $\lambda = 0.0104$. Then putting that back into the decay equation, we get $Q(t) = Q(0)e^{-0.0104t}$. We want t so that $\frac{Q(t)}{Q(0)} = 0.35$, so $0.35 = e^{-0.0104t}$. Solving this exponential equation we get $\ln(0.35) = -1.049 = -0.0104t$, so $t = 100.9$ years for 65% to decay or 35% to remain.