

FALL 2026 - CALCULUS 2 - TEST 2A - Solutions

True or false:

True 1) Differential equations involve one or more derivatives

True 2) Separable equations are a type of differential equation

False 3) Solutions of separable equations always lead to exponential growth - could be decay

True 4) $\frac{dy}{dx} = 4x$ is a separable equation - $dy = 4xdx$

True 5) $\frac{dy}{dx} = 4y$ is a separable equation - $\frac{dy}{y} = 4dx$

True 6) $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$

True 7) $\frac{dy}{dx} = 4 \sin x \cos y$ is a separable equation - $\frac{dy}{\cos y} = 4 \sin x dx$

True 8) $-\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = -\frac{\cosh x}{\sinh x}$

False 9) $\frac{dy}{dx} = 4 \sin xy$ is a separable equation - $\sin xy$ can't be broken into two separate functions

False 10) $\frac{dy}{dx} = 4e^{xy}$ is a separable equation - $e^{xy} = (e^x)^y$ not $e^{x+y} = e^x \cdot e^y$

True 11) Carbon-14 dating is based on radioactive decay

False 12) The natural logarithm is defined in terms of the area under the curve $y = e^x$ - under hyperbola $y = \frac{1}{x}$

True 13) $\ln x = \int_1^x \frac{1}{t} dt$

True 14) logarithmic growth is always faster than polynomial growth

False 15) If $f(x) = o(g(x))$, then $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$ - $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$

False 16) If $f(x) = O(g(x))$, then $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$ - $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = C > 0$

False 17) If $f(x) = O(g(x))$, then $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \infty$ - $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = C > 0$

False 18) $x^3 = O(x^2 + x)$ - $\lim_{x \rightarrow \infty} \frac{x^2 + x}{x^3} = \lim_{x \rightarrow \infty} \left(\frac{1}{x} + \frac{1}{x^2} \right) = 0$

True 19) $e^x = o(3^x)$ - $\lim_{x \rightarrow \infty} \frac{2.71828^x}{3^x} = \lim_{x \rightarrow \infty} \left(\frac{2.71828}{3} \right)^x = \lim_{x \rightarrow \infty} \alpha^x = 0$, since $\alpha < 1$

False 20) $e^x = O(2^x)$ - $\lim_{x \rightarrow \infty} \frac{2.71828^x}{2^x} = \lim_{x \rightarrow \infty} \left(\frac{2.71828}{2} \right)^x = \lim_{x \rightarrow \infty} \alpha^x = \infty$, since $\alpha > 1$

False 21) $\frac{x^4 + 5x^3 - 2x^2 + 4x + 2}{x^2 + 7x - 5} = o(x^2)$ - $O(x^2)$, divide top & bottom by x^2

True 22) $Q(t) = Q(0)e^{kt}$ could model radioactive decay - if $k < 0$

True 23) $P(t) = P(0)e^{rt}$ could model continuous interest - the PERT formula

False 24) $\frac{dy}{dt} = kt$ could model heating/cooling if y is temperature difference - $\frac{dy}{dt} = ky$

False 25) $\frac{dy}{dx} = -kx^2$ could model light intensity decay through an absorbing medium - $\frac{dy}{dx} = ky$, where x is distance