Logic - Lecture 21

Invalidity in Relational Logic

Nothing conceptually different from what we have been doing in single quantifier predicate logic. Both the natural interpretation method and model universe method can be applied. Recall for the natural interpretation scheme we try to find meanings (endow with semantic content) for predicate letters and individual constants in such a way that a counterexample emerges. For example, suppose we argue

$(y)(\exists x)Fxy \; / \therefore (\exists x)(y)Fxy$.

We already suspect this is invalid based on our discussion of the commutation relations among multiple quantifiers. Here is a natural interpretation which demonstrates invalidity:

Let Fxy mean "x is the biological father of y". Then the single premise asserts "for all y, there exists an x that is the biological father of y"... or "everyone has a father." OTOH, the conclusion asserts that "for some x, and for all y, x is the father of y"... or "one person is the father of everyone."

That is quite a different claim. And the counterexample shows that the argument is invalid.

Whenever individual constants appear in the argument, we have to give them an interpretation as well. Consider:

① $(\exists x)Fxa$, ② $(x)Fxa \supset (\exists x)Gax$ / $\therefore (\exists x)Gax$. Let the domain from which we can choose individuals be all human beings, past or present. Then let Fxy mean "x is the biological father of y", Gxy mean "x is the biological mother of y", and

A

a = "Albert Einstein". Then premise ① says "Albert Einstein had a father." Premise ② says "if everyone is the father of Albert Einstein, then Albert Einstein is someone's mother." Both of these premises are true, ② because it is a material implication where the antecedent is false (F implies F is true!). However the conclusion claims that Einstein is somebody's mother, which we are fairly certain is false.

The model universe method is applied just as before. Revisiting the argument $(y)(\exists x)Fxy \;/\therefore (\exists x)(y)Fxy$, we set up a universe of two elements: $U = \{a, b\}$. The leftmost quantifier in $(y)(\exists x)Fxy$ is universal, so we can apply it to the universe and write $(\exists x)Fxa \cdot (\exists x)Fxb$. Going further, we rewrite the existential quantifications as disjunctions, so

$(\exists x)Fxa \equiv (Faa \vee Fba)$ and

$(\exists x)Fxb \equiv (Fab \vee Fbb)$. Putting it all together, we have

$(Faa \vee Fba) \cdot (Fab \vee Fbb)$. Likewise the conclusion can be written

$/ \therefore (Faa \cdot Fab) \vee (Fba \cdot Fbb)$.

Now if we make Faa and Fbb false

but Fab and Fba true, we have the premise

$$(Faa \vee Fba) \cdot (Fab \vee Fbb)$$
$$(F \vee T) \cdot (T \vee F)$$
$$T \cdot T$$
$$T$$

but the conclusion

$$(Faa \cdot Fab) \vee (Fba \cdot Fbb)$$
$$(F \cdot T) \vee (T \cdot F)$$
$$F \vee F$$
$$F$$

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So we have constructed a counterexample.