Logic - Lecture 20

Relational Predicate Proofs

Remember the argument:

1) Some people don't like any dogs

2) All puppies are dogs

/∴ Some people don't like puppies

With our relational machinery we can symbolize this as:

P person	U puppy
D dog	L likes

1) $(\exists x)(Px \cdot (y)(Dy \supset \neg Lxy))$

2) $(x)(Ux \supset Dx)$

$/\therefore (\exists x)(Px \cdot (y)(Uy \supset \neg Lxy))$

Continuing with the formal proof:

3) $Pa \cdot (y)(Dy \supset \neg Lay)$ EI on 1; flag a

4) Pa 3, Simp

b/x means substitute b for x

5) $(y)(Dy \supset \neg Lay)$ 3, Simp

6) flag b FS for UG

7) $Ub \supset Db$ UI on 2, b/x

8) $Db \supset \neg Lab$ UI on 5, b/y

9) $Ub \supset \neg Lab$ 7,8; HS

10) $(y)(Uy \supset \neg Lay)$ UG on 9, y/b

11) $Pa \cdot (y)(Uy \supset \neg Lay)$ 4,10; Conj

12) $(\exists x)(Px \cdot (y)(Uy \supset \neg Lxy))$ EG on 11, x/a

A few observations: All the inference principles and replacement rules are available to us. Remember to be consistent and replace <u>every</u> occurrence of variables or constants when you do a substitution in the course of an instantiation or generalization. Also recall that you can't use instantiation rules on quantifiers in the middle of a formula. For example, in the above proof we simplified the first premise to obtain line (5), which was in quantifier form. The same limitations apply to generalization. Here are some typical errors, so you can see what to avoid:

1. $(x)(Fx \supset (y)(Gy \supset Hxy))$

$/\therefore (x)(Fx \supset (Gb \supset Hxb)$ by UI using b for y – Error

2 $(\exists x)(Fx \cdot (\exists y)(Hxy \cdot Jyx)$

$/\therefore (\exists x)(Fx \cdot Hxa \cdot Jax)$ by EI

using a for y - Error

3 $Fa \supset (y)(Gy \supset Hay)$

$/\therefore Fa \supset (Gb \supset Hab)$ by UI using

b for y - Error

4 $(x)(Fx \supset (Gax \cdot Hax))$

$/\therefore (x)(Fx \supset (\exists y)(Gyx \cdot Hyx))$ by EG

using y for a - Error.

It's a good idea to avoid using variable letters as instance letters. This could cause a lot of confusion. Also, in proofs where you may need two or more flagged letters, make a conscious effort to not use the same letter twice.

Here is an example of the correct approach.

1) $(x)(Px \supset (\exists y)Fyx)$

2) $(x)(y)(Fyx \supset Lyx)$

/∴ $(x)(Px \supset (\exists y)Lyx)$

3) $\underline{\underline{\text{flag } a}}$ FS for UG

4) Pa CP assumption

5) $Pa \supset (\exists y)Fya$ UI on 1, a/x

6) $(\exists y)Fya$ 4,5; MP same!

7) Fba EI on 6, b/y $\underline{\text{flag } b}$

8) $(y)(Fya \supset Lya)$ UI on 2, a/x

9) $Fba \supset Lba$ UI on 8, b/y

10) Lba 7,9; MP

11) $\underline{(\exists y)Lya}$ EG on 10 y/b

12) $Pa \supset (\exists y)Lya$ CP 4 to 11

13) $(x)(Px \supset (\exists y)Lyx)$ UG on 12 x/a

A strategy for these proofs is to always work from both ends to the middle. Ask yourself what you would need for a final instantiation or generalization to get the conclusion. In this case we knew if we could deduce $Pa \supset (\exists y)Lya$ and then generalize a, we would have it. So then our attention turns to getting that from the premises. Since this statement was a conditional, the method of conditional proof naturally suggested itself.

Remember:

1. A letter being flagged must be new to the proof. It cannot have appeared in any previous formula or as a letter having been flagged before.
2. A flagged letter never appears in the premises or conclusion
3. A flagged letter never appears outside the subproof where it is introduced.