Logic - Lecture 19

Quantifier Negation

The same rules as before apply to multiply quantified statements. It is important to note that you can apply the negation rules to only one quantifier at a time.

For example:

① $\neg(\exists x)(\exists y)\neg Fxy \equiv (x)\neg(\exists y)\neg Fxy$

② $\neg(x)(y)\neg Fxy \equiv (\exists x)\neg(y)\neg Fxy$

③ $(\exists x)\neg(y)\neg Fxy \equiv \neg(x)(y)\neg Fxy$

④ $(x)\neg(\exists y)\neg Fxy \equiv \neg(\exists x)(\exists y)\neg Fxy$

Also keep in mind the replacement rules can be applied to subformulas as well.

So the right hand sides above could be written as:

1. $(x)(y) \neg\neg Fxy$
2. $(\exists x)(\exists y) \neg\neg Fxy$
3. $\neg(x)\neg(\exists y) Fxy$
4. $\neg(\exists x)\neg(y) Fxy$

More examples:

1. $\neg(\exists x)\neg(y) Fxy \equiv (x)(y) Fxy$
2. $\neg(x)\neg\neg(\exists y) Fxy \equiv (\exists x)\neg(\exists y) Fxy$
3. $\neg(x)\neg(\exists y)\neg Fxy \equiv \neg(x)(y) Fxy$

   or $\equiv (\exists x)(\exists y)\neg Fxy$
4. $\neg(\exists x)\neg(y)\neg Fxy \equiv \neg(\exists x)(\exists y) Fxy$

   or $\equiv (x)(y)\neg Fxy$
5. $(x)(y) Fxy \equiv (x)\neg(\exists y)\neg Fxy$

If you use the QN rules several times, you can "run through" a negation:

1. $\neg(x)(y)(z)Fxyz \equiv (\exists x)(\exists y)(\exists z)\neg Fxyz$
2. $\neg(\exists x)(y)(\exists z)Fxyz \equiv (x)(\exists y)(z)\neg Fxyz$

and so forth.

Here are some examples both symbolically and with English equivalents:

1. No one introduced anybody to Richard.

Same as: For everyone, they introduced no one to Richard. Using $Ixyz$ for "x introduced y to z", we have these two statements symbolically as $\neg(\exists x)(\exists y)Ixyr$

r = "Richard" and $(x)\neg(\exists y)Ixyr$, respectively

2. Not everybody introduced someone

j = "John" to John. Same as: Some people introduced no one to John. Symbolically:

First statement: $\neg (x)(\exists y) Ixyj$

Second statement: $(\exists x) \neg (\exists y) Ixyj$

or $(\exists x)(y) \neg Ixyj$

3 No one introduced anybody to anyone.
$[\neg (\exists x)(\exists y)(\exists z) Ixyz]$

Same as: For everyone, they were not introduced to anyone. $[(x) \neg (\exists y)(\exists z) Iyxz]$

Same as: For every x and y, no one introduced x to y. $[(x)(y) \neg (\exists z) Izxy$

Symbolization

Here is a method that gradually arrives at a proper symbolization of the sort of relational statements we now are studying.

Problem: Symbolize

English: "There is a dog who catches mice and his owner doesn't feed him."

1st pass: $(\exists x)$(x is a dog · x has owner who doesn't feed him · x catches mice)

2nd pass: $(\exists x)(Dx \cdot (\exists y)$(y owns x any y doesn't feed x) · (x catches mice))

3rd pass: $(\exists x)(Dx \cdot (\exists y)(Oyx \cdot \neg Fyx) \cdot$ (x catches mice))

4th pass: $(\exists x)(Dx \cdot (\exists y)(Oyx \cdot \neg Fyx) \cdot (\exists z)$(z is mouse and x catches z))

Final: $(\exists x)(Dx \cdot (\exists y)(Oyx \cdot \neg Fyx) \cdot (\exists z)(Mz \cdot Cxz))$

Rule of Thumb: just like single quantifier logic, universal formulas almost always have $\supset$ as the major operator and existential formulas almost always have $\cdot$ (conj) as the major operator. If your symbolizations vary from this pattern, they are likely incorrect.

Problem: Symbolize:

"Not all students enjoy all of their courses."

The form of this sentence is a negated universal where the subject is "students" and the predicate is "things that enjoy their courses".

So the general structure will be

$\neg(x)(\_\_\_\_ \supset \_\_\_\_)$. Paraphrasing, "It is not the case that for every x if x is a student then x enjoys all of x's courses". We could symbolize the predicate here (with obvious abbrev)

$$(y)((Cy \cdot Txy) \supset Exy)$$

course   x takes y   x enjoys y

Then overall we have:

$$\neg(x)(Sx \supset (y)((Cy \cdot Txy) \supset Exy))$$

Convert that to an existential for practice.. Final advice: all variables should be bound in the final symbolic representation.