Logic - Lecture 18

Relational Predicate Logic

We have seen that there are arguments that can't be resolved by sentential logic - we need quantifier logic for them. Likewise, there are arguments that are clearly valid, but which cannot be established using quantifier logic:

1) Some people don't like any dogs
2) All puppies are dogs
/∴ Some people don't like puppies

We could try to show this is valid using predicate logic methods.

Let Px = x is a person
Lx = x likes dogs
Ux = x is a puppy
Dx = x is a dog
Kx = x likes puppies ... then

1) $(\exists x)(Px \cdot \neg Lx)$

2) $(x)(Ux \supset Dx)$

/ $\therefore$ $(\exists x)(Px \cdot \neg Kx)$

Try to give a proof. There is no way to get a handle on $\neg Kx$. Yet the argument is perfectly valid.

We need an upgrade to quantifier logic - relational predicate logic.

A polyadic predicate states a relationship between two or more individuals. In regular predicate logic we just asserted something had property P by writing Px. Now we want to expand this so that "x loves y" becomes Lxy and "x is taller than y" becomes Txy and so forth. Order of appearance of variables after the predicate letter is crucial. "x is between y and z" can be symbolized as Bxyz ... not Byxz, which does not conform to the order of appearance of variables in the statement (although on the page it is true but irrelevant).

Here are some more examples:

1) John loves Mary   Ljm
   x loves y   Lxy

2) Andrew works for Joe   Waj
   x works for y

3) Texas is south of Iowa   Sti
   x is south of y

4) Rockefeller owns Citibank   Orc
   x owns y

5) Richard Burton gave Elizabeth Taylor the Hope Diamond   Grhe

   x gave y to z   Gxyz

6) Elizabeth Taylor gave George the Bulgarian Emerald in exchange for the Hope Diamond   Gebgh

x gave y to z in exchange for w

Gxyzw

Now we introduce quantifiers for these predicates.

Going back to Lxy... x loves y and j = John:

$(\exists x)Lxj$ means someone loves John

$(x)Lxj$ means everyone loves John

$(\exists x)Ljx$ means John loves someone

$(x)Ljx$ means John loves everyone

We can have multiple quantifiers.

$(x)(y)Lxy$ "every x loves every y"

$(y)(x)Lxy$ "every y is loved by every x"

$(\exists x)(\exists y)Lxy$ "some x loves some y"

$(\exists y)(\exists x)Lxy$ "some y is loved by some x"

Note that $(x)(y)Lxy \equiv (y)(x)Lxy$

This is due to the commutativity of conjunction. Also..

$(\exists x)(\exists y)Lxy \equiv (\exists y)(\exists x)Lxy$.

This is related to the commutativity of disjunction.

However if we <u>mix</u> types of quantifiers, things change.

What does $(x)(\exists y)Lxy$ mean?

For every x, there is some y ...

(x) $(\exists y)$

... such that x loves y.

Lxy

In short, "for every x, x loves somebody" or "everybody loves somebody".

What does $(\exists y)(x)\,Lxy$ mean?

There is some y such that ...
$(\exists y)$

for all x ... x loves y
(x) Lxy

In short, "there is some y that every x loves" or "there is someone whom everyone loves."

You can work out $(\exists x)(y)\,Lxy$ and $(y)(\exists x)\,Lxy$.

Now let $Ixyz$ = "x introduced y to z" and j = John, m = Mary.

$(\exists x)(\exists y)\,Ixyj$ means "someone introduced somebody to John"

$(z)(\exists y) Imyz$ means "Mary introduced someone to everyone."

Try to paraphrase:

(i) $(\exists x)(\exists y) Ixmy$

(ii) $(\exists y)(z) Imyz$

<u>Note</u>: the leftmost quantifier determines the form of the sentence.

Also, do you see why

$(\exists x)(y) Lxy$ and $(\exists y)(x) Lyx$

are really the same? The bound variables in each have been consistently interchanged. Next we will discuss quantifier negation with relational predicates.