Logic - Lecture 17

Invalidity in Predicate Logic

Just because you can't find a proof doesn't mean a proof doesn't exist. We need a more dispositive way of determining invalidity in the context of quantifier logic. There are two methods:

Natural Interpretation:

A counterexample in predicate logic is the same as we saw in sentential logic. We determine the form of an argument and try to construct an instance of the form where all premises are true and the conclusion is false. The so-called natural interpretation method involves assigning meaning to the predicate letters so that we develop true premises but a false conclusion. For example:

1) All communists are in favor of socialized medicine.

2) All socialists are in favor of socialized medicine.

3) Therefore, all socialists are communists.

Symbolically,

1) $(x)(Cx \supset Fx)$

2) $(x)(Sx \supset Fx)$

/ $\therefore$ $(x)(Sx \supset Cx)$

OK, now let Cx = x is a normal man, Sx = x is a normal woman, Fx = x has a brain. Using <u>these</u> meanings the argument says:

1) All normal men have a brain
2) All normal women have a brain
∴ All normal women are normal ~~wo~~men

Interpreted this way, we see immediately that the original argument is invalid.

So the S.O.P. is

1) State the domain of discourse
2) Interpret all predicate letters by assigning some English predicate to each (here English means non-symbolic)

3) Write out the fully interpreted English sentences and find a counterexample with all your premises true.

Example: Show (1) $(x)(Ax \supset Bx)$, (2) $(\exists x)(Ax \cdot Cx)$, (3) $/\therefore (x)(Cx \supset Bx)$ is invalid.

1) Domain ≡ entire universe

2) Ax = x is a cat
Bx = x is a mammal
Cx = x has four legs

3) All cats are mammals
Some cats have four legs
$/\therefore$ All four-legged things are mammals

Nope, how about a dining room table?!

Model Universe Method

Here we make use of the fact that all it takes to make an argument invalid is to find some universe of discourse where it is false. Here is the idea:

Show (1) $(\exists x)(Fx \cdot Gx)$, (2) $(\exists x)(Gx \cdot Hx)$, (3) $/\therefore (\exists x)(Fx \cdot Hx)$ is invalid.

A1) Let the domain be restricted to two members $\{a, b\}$.

A2) Then statement (1) in this context is $(Fa \cdot Ga) \vee (Fb \cdot Gb)$

A3) Statement (2) says $(Ga \cdot Ha) \vee (Gb \cdot Hb)$

A4) The conclusion says $(Fa \cdot Ha) \vee (Fb \cdot Hb)$

A5) Now assign Fa = T, Ga = T, Ha = F,
Fb = F, Gb = T, and Hb = T.

A6) Then (1) is true since (Fa · Ga) is true.
Also (2) is true since (Gb · Hb) is true.
Finally (3) is false because *both*
(Fa · Ha) and (Fb · Hb) are false.

So the trick for the model universe is to make it *small* and engineer the truth value assignments to invalidate the conclusion while keeping the premises true. If the argument contains a constant originally, don't forget to put that constant in the universe.