Logic - Lecture 16

We now add the last level of complexity to our predicate logic proofs. Previously we dealt with statements in quantifier form as premises. The typical procedure was to drop quantifiers to get an instance, manipulate that with our inference and replacement rules, then pass back to a conclusion in quantifier form. We would like to broaden what we can handle and allow truth-functional compounds of quantified statements. Suppose we are given the premise $(x)(Fx \supset Gx)$ - which recognize as being in quantifier form - and $(x) Gx \supset (x) Hx$, which definitely is not. The desired conclusion is also not in quantifier form: $(x) Fx \supset (x) Hx$. We could begin our argument with $Fa \supset Ga$ from UI on premise 1, but it would be invalid to get $Ga \supset Ha$ from premise 2. Premise 2 is not a universal formula... it is a conditional and UI on premise 2 does not work... i.e. $Ga \supset Ha$ would require instantiating a on both sides of the $\supset$. Even if we had $Fa \supset Ha$, we could not use UG to recover

$(x)Fx \supset (x)Hx$.

So how do we do this? Set up the premises and work backwards from the conclusion. What would give us $(x)Fx \supset (x)Hx$. Well, a conditional proof would, where we assume $(x)F(x)$ and eventually conclude $(x)Hx$. So...

1) $(x)(Fx \supset Gx)$ Premise

2) $(x)(Gx \supset (x)Hx$ Premise

/ $\therefore (x)Fx \supset (x)Hx$ Conclusion

3) $(x)Fx$ Assumption for CP

⋮

n) $(x)Hx$

n+1) $(x)Fx \supset (x)Hx$ 3 thru n, CP

OK, that looks like it will do it if we can supply the CP steps.

Now everything:

1) $(x)(Fx \supset Gx)$ Premise

2) $(x)Gx \supset (x)Hx$ Premise

/ $\therefore$ $(x)Fx \supset (x)Hx$ Conclusion

3) $(x)Fx$ — Assumption for CP

4) flag a — Flagging for UG

5) Fa — 3; UI

6) $Fa \supset Ga$ — 1; UI — why can we use a here?

7) Ga — 5,6; MP

8) $(x)Gx$ — 7; UG

9) $(x)Hx$ — 2,8; MP

10) $(x)Fx \supset (x)Hx$ — 3 to 9; CP

Primary rule to stay out of trouble is to use UI, UG, EI, & EG only on statements in quantifier form.

Here is a predicate logic analog of something we did in sentential logic.

A predicate logic theorem is a formula that is always true and has no premises.

For example:

$$(\exists x)(Fx \cdot Gx) \supset [(\exists x)Fx \cdot (\exists x)Gx]$$

1)	$(\exists x)(Fx \cdot Gx)$	Assumption for CP
2)	$Fa \cdot Ga$	1; EI / flag a
3)	Fa	2; Simp
4)	Ga	2; Simp
5)	$(\exists x)Fx$	3; EG

⋮

6) $(\exists x)Gx$ 4; EG

7) $(\exists x)Fx \cdot (\exists x)Gx$ 5, 6; Conj

8) $(\exists x)(Fx \cdot Gx) \supset [(\exists x)Fx \cdot (\exists x)Gx]$

1 to 7; CP

Don't forget you can use a proof by contradiction inside one of these predicate logic proofs.

SUMMARY OF QUANTIFIER RULES

Preliminaries:

1) ϕx is a propositional function on x, simple or complex. If complex, it is assumed that it is enclosed in parentheses, so that the scope of any prefixed quantifier extend to the end of the formula.

2) ϕa is a formula just like ϕx, except every occurrence of x in ϕx has been replaced by an a.

3) An **instance** of a general formula is the result of deleting the initial quantifier and replacing each variable bound by that quantifier uniformly with some name (such as a).

4) An a-**flagged subproof** is a subproof that begins with the words "*flag* a" and ends with some instance containing a.

The Four Quantifier Rules:

1) Universal Instantiation (**UI**)
$(x)\phi x$, therefore ϕa

2) Existential Instantiation (**EI**)
$(\exists x)\phi x$, therefore ϕa, provided a is flagged

3) Universal Generalization (**UG**)
execute a-flagged subproof to get ϕa, therefore $(x)\phi x$

4) Existential Generalization (**EG**)
ϕa, therefore $(\exists x)\phi x$

How flags work:

1) A letter being flagged must be new to the proof...it must not appear in a preceding formula or other flagging step

2) A flagged letter must never appear in a premise or conclusion

3) A flagged letter must not appear outside the subproof in which it is flagged