B7LEC

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Jacobians:

$x = x(u,v) \quad y = y(u,v)$

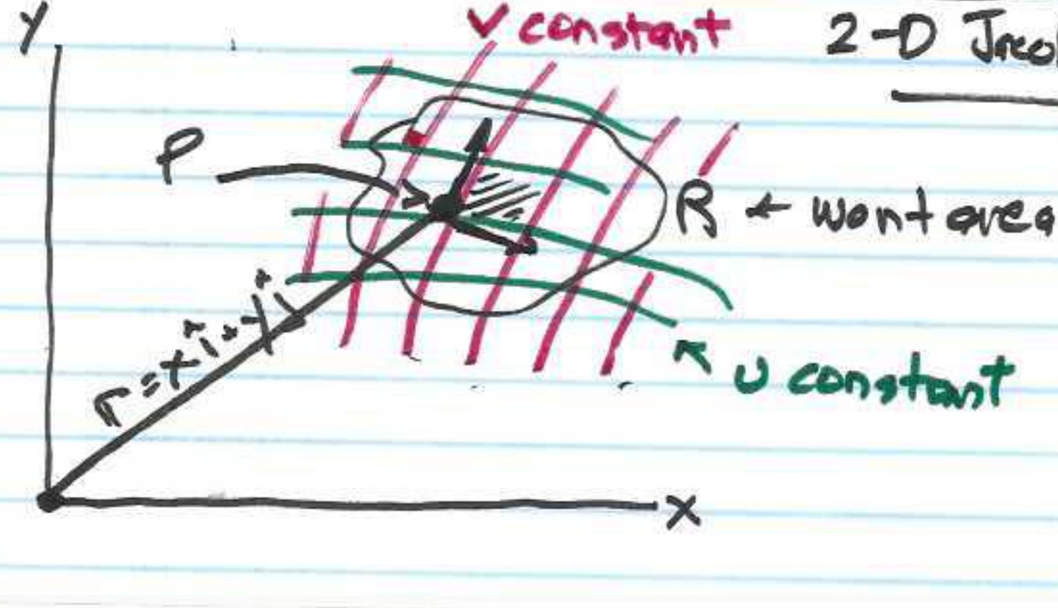
$$\frac{\partial(x,y)}{\partial(u,v)}$$

If $\frac{\partial(x,y)}{\partial(u,v)} \neq 0$ @ point P, then

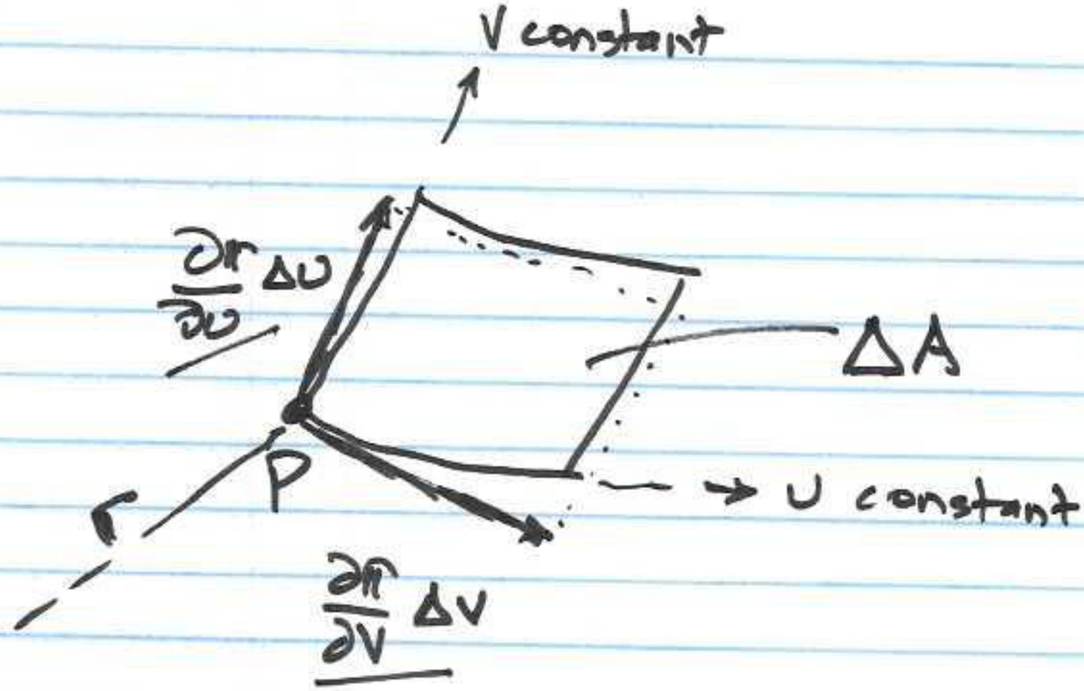
$$\frac{\partial(u,v)}{\partial(x,y)} = \left(\frac{\partial(x,y)}{\partial(u,v)}\right)^{-1} \leftarrow \quad f'(x) = \frac{1}{x'(f)} \leftarrow \neq 0$$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} \leftarrow \neq 0$$

2-D Jacobian



②



$$\Delta A \approx \left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right| \Delta u \Delta v \not\approx \Delta x \Delta y$$

$$\partial A = \left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right| du\,dv = dx\,dy$$

$$\mathbf{r} = x\hat{i} + y\hat{j}$$

$$x = r\cos\theta \; ; \; y = r\sin\theta \; ; \; u = r, \; v = \theta$$

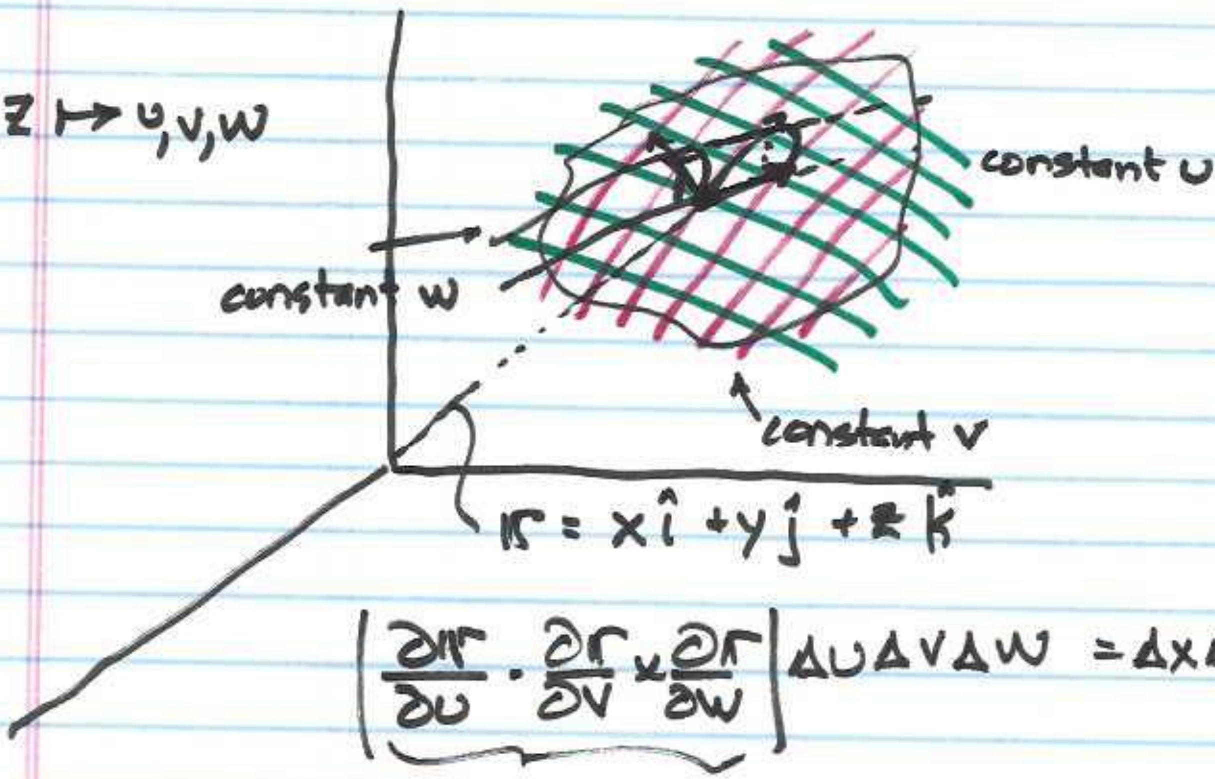
$$\begin{vmatrix} \frac{\partial(r\cos\theta)}{\partial r} & \frac{\partial(r\sin\theta)}{\partial r} & 0 \\ \frac{\partial(r\cos\theta)}{\partial\theta} & \frac{\partial(r\sin\theta)}{\partial\theta} & \\ 0 & 0 & 1 \end{vmatrix}$$

$(A\times B)_k$ 3 $\underline{A\cdot B\times C}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = (r\cos^2\theta + r\sin^2\theta)\,\hat{k}$$

$$\mathbf{r} = \frac{\partial(x,y)}{\partial(r,\theta)} \quad \Bigg| \quad \frac{\partial(r,\theta)}{\partial(x,y)} = \underline{r^{-1}}$$

$x,y,z \mapsto u,v,w$



$$\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

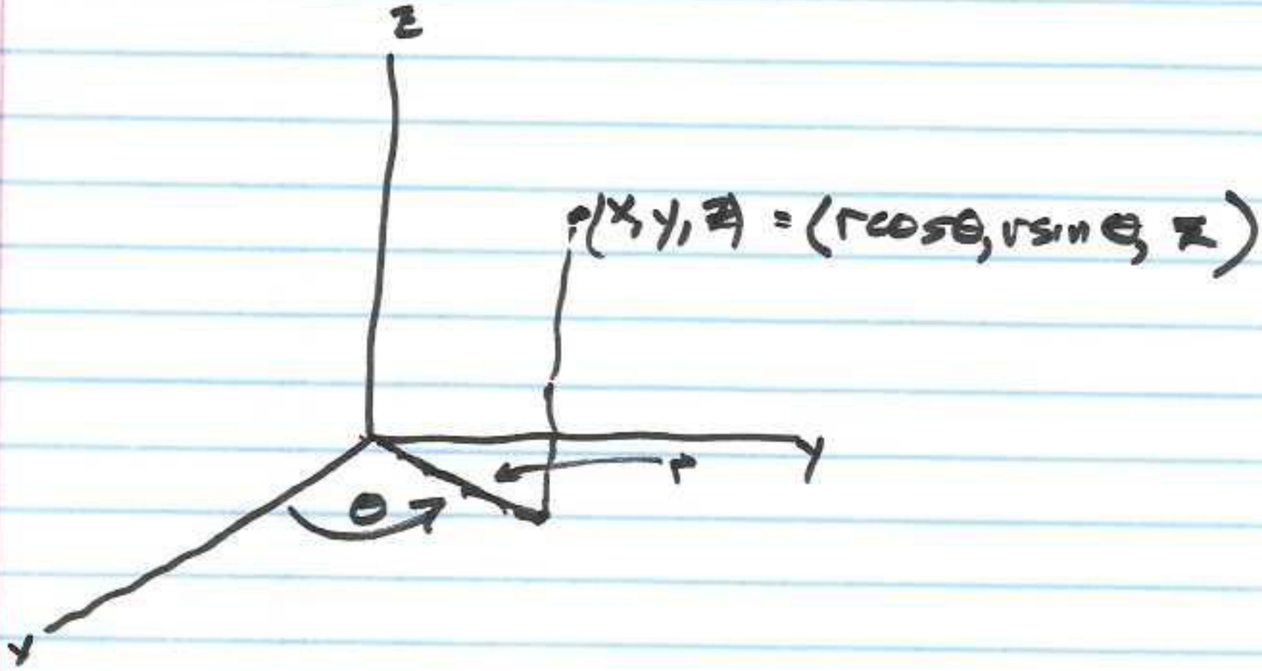
$$\left|\frac{\partial \mathbf{r}}{\partial u}\cdot\frac{\partial \mathbf{r}}{\partial v}\times\frac{\partial \mathbf{r}}{\partial w}\right| \Delta u\Delta v\Delta w = \Delta x\Delta y\Delta z$$

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial w} & \frac{\partial \cancel{x} y}{\partial w} & \frac{\partial z}{\partial w} \end{vmatrix} = ?$$

Circular Cylindrical Co-ords:

$(x, y, z) \to (r, \theta, z')$

$x = r\cos\theta$; $y = r\sin\theta$; $z = z'$

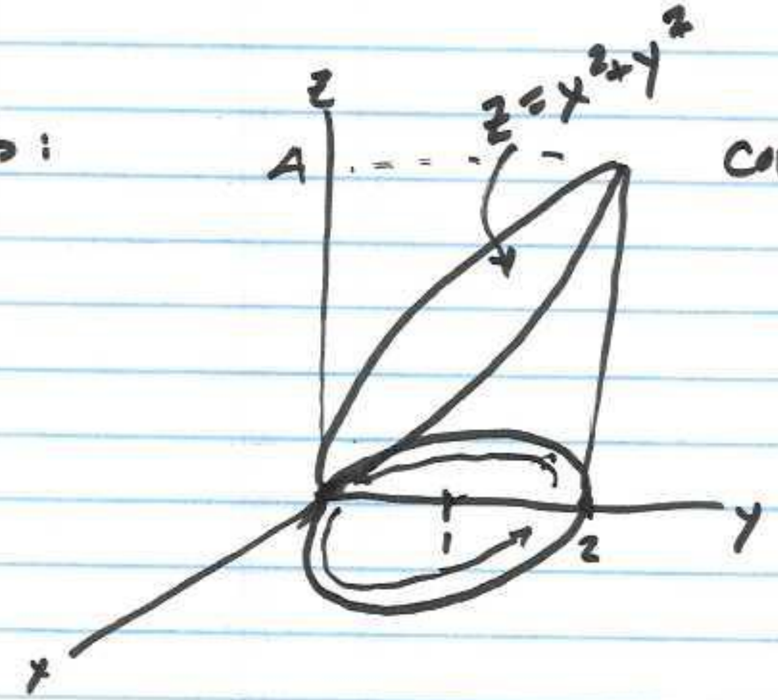


⑤

volume adjustment

$$\bigstar \quad \iiint f(x(r,\theta,z'), y(r,\theta,z'), z(r,\theta,z'))\, \underbrace{r}_{\text{volume adjustment}}\, dr\, d\theta\, dz$$

Prob:



cartesian $\underbrace{x^2 + (y-1)^2 = 1}$

$$r^2\cos^2\theta + (r\sin\theta - 1)^2 = 1$$

$$\underbrace{r^2\cos^2\theta + r^2\sin^2\theta} - 2r\sin\theta + 1 = 1$$

$$r^2 - 2r\sin\theta = 0$$

$$\cancel{r}^2 = 2\cancel{r}\sin\theta$$

$$\underline{r = 2\sin\theta \text{ is floor}}$$

(6)

$$z = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta) = r^2$$

$z = r^2$ (roof)

$$\iiint_R 1\,dx\,dy\,dz = \int\int\int (1)\, r\,dr\,d\theta\,dz$$

$$\text{vol} = \int_0^{\pi}\int_0^{2\sin\theta}\int_0^{r^2} (1)(r)\,\underline{dz\,dr\,d\theta}$$

$$= \int_0^{\pi}\int_0^{2\sin\theta} \left[z\right]_0^{r^2} \cdot r\,dr\,d\theta$$

$$= \int_0^{\pi}\int_0^{2\sin\theta} r^3\,dr\,d\theta = \int_0^{\pi} \left[\frac{r^4}{4}\right]_0^{2\sin\theta} d\theta$$

$$= 4\int_0^{\pi} \sin^4\theta\,d\theta \overset{?}{=}$$

from table: $$\int \sin^4\theta\,d\theta = -\frac{\sin^3\theta\cos\theta}{4} + \frac{3}{4}\underbrace{\int \sin^2\theta\,d\theta}_{\frac{\theta}{2} - \frac{\sin 2\theta}{4}}$$

$$\text{Vol} = 4\left[\frac{-\sin^3\theta\cos\theta}{4} + \frac{3}{4}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right]_0^{\pi}$$

no contribution

no contribution

$$= \cancel{4}\left(\frac{3}{\cancel{4}}\right)\left(\frac{\pi}{2}\right) = \frac{3\pi}{2}$$

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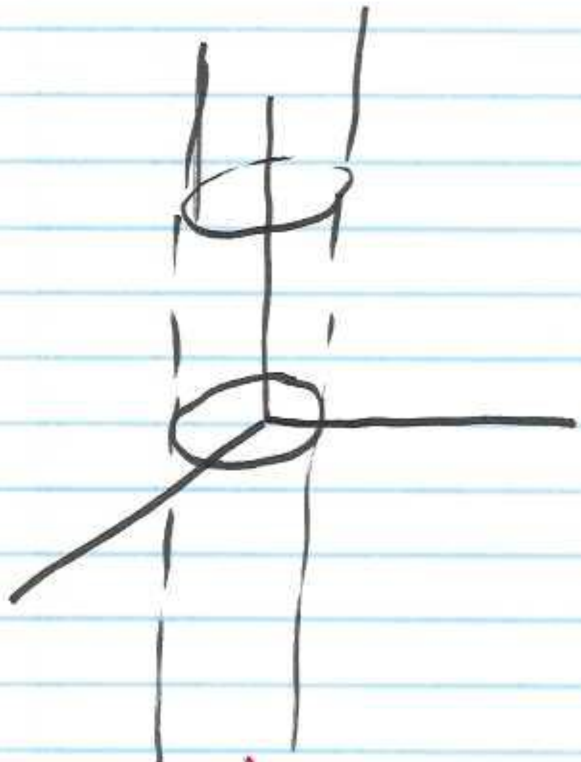
1 $r = 2$



2 $\theta = \frac{\pi}{4}$