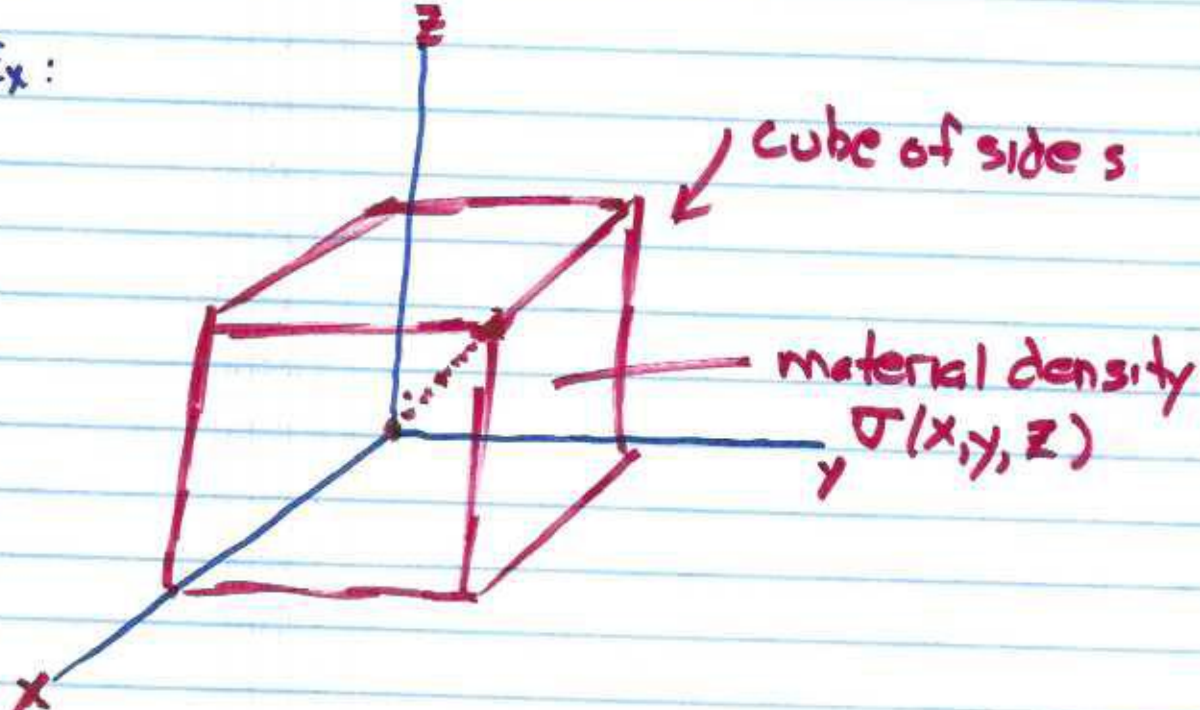
FPQGN

1

3/27

Center of Mass / Centroid / MOI (inertia)

Ex:



CM has co-ordinates $(\bar{x}, \bar{y}, \bar{z})$

$$\bar{x} := \frac{M_{yz}}{M} = \frac{\iiint x\,\sigma(x,y,z)\,\underbrace{dx\,dy\,dz}_{dq^3}}{\iiint \sigma(x,y,z)\,dx\,dy\,dz}$$

$$\bar{y} := \frac{M_{xz}}{M} = \frac{\iiint y\,\sigma(x,y,z)\,dx\,dy\,dz}{M}$$

②

$$\bar{z} := \frac{M_{xy}}{M} = \frac{\iiint z\,\sigma(x,y,z)\,dx\,dy\,dz}{M}$$

$$M := \iiint dm$$

$$dm = \sigma(x,y,z)\,dx\,dy\,dz$$

Go back to cube of side s and give it

$$\sigma = (1 + x^2 + y^2 + z^2)$$

$$M_{yz} = \int_0^s \int_0^s \int_0^s x\,(1 + x^2 + y^2 + z^2)\,dx\,dy\,dz$$

$$= \int_0^s \int_0^s \int_0^s (x + x^3 + xy^2 + xz^2)\,dx\,dy\,dz$$

$$= \int_0^s \int_0^s \left[\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^2z^2}{2}\right]_0^s dy\,dz$$

(3)

$$= \int_0^s \int_0^s \left(\frac{s^2}{2} + \frac{s^4}{4} + \frac{s^2 y^2}{2} + \frac{s^2 z^2}{2} \right) dy\, dz$$

$$= \int_0^s \left[\frac{s^2 y}{2} + \frac{s^4}{4} y + \frac{s^2 y^3}{6} + \frac{s^2 y z^2}{2} \right]_0^s dz$$

$$= \int_0^s \left(\frac{s^3}{2} + \frac{s^5}{4} + \frac{s^5}{6} + \frac{s^3 z^2}{2} \right) dz$$

$$= \int_0^s \left(\frac{s^3}{2} + \frac{5}{12} s^5 + \frac{s^3}{2} z^2 \right) dz$$

$$= \left(\frac{s^3 z}{2} + \frac{5 s^5 z}{12} + \frac{s^3 z^3}{6} \right) \Big|_0^s$$

$$= \frac{s^4}{2} + \frac{5}{12} s^6 + \frac{s^6}{6} = \underline{s^4 \left[\frac{1}{2} + s^2 \cdot \frac{7}{12} \right]}$$

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$$\sigma(x,y,z)$$

$$M = \int_0^s \int_0^s \int_0^s \left(1 + x^2 + y^2 + z^2\right) dx\,dy\,dz$$

$$= \int_0^s \int_0^s \left[x + \frac{x^3}{3} + y^2 x + z^2 x\right]_0^s dy\,dz$$

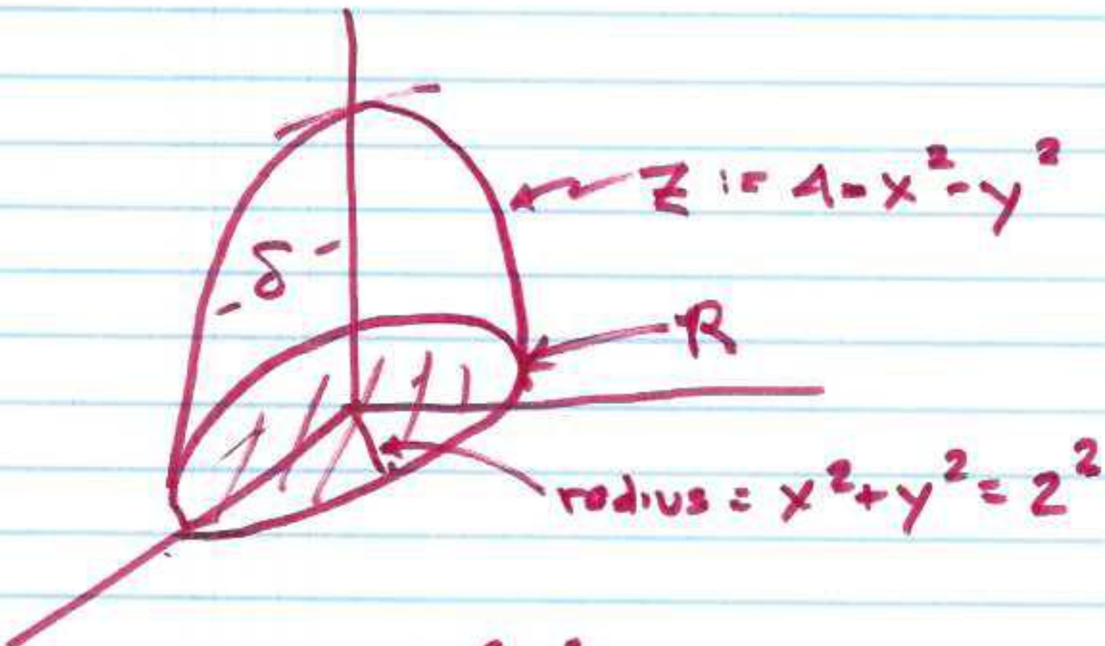
$$= \int_0^s \int_0^s \left(s + \frac{s^3}{3} + s y^2 + s z^2\right) dy\,dz$$

$$= \int_0^s \left[sy + \frac{s^3}{3} y + \frac{s y^3}{3} + s y z^2\right]_0^s dz$$

$$= \int_0^s \left(s^2 + \underline{\frac{s^4}{3} + \frac{s^4}{3}} + s^2 z^2\right) dz$$

$$= \left[s^2 z + \frac{2}{3} s^4 z + \frac{s^2 z^3}{3}\right]_0^s$$

Ex (p.933) δ = constant



$$M_{xy} = \iint_R \int_{z=0}^{4-x^2-y^2} z\,\delta\,dz\,dy\,dx =$$

$$\delta \iint_R \left[\frac{z^2}{2}\right]_0^{4-x^2-y^2} dy\,dx$$

$$= \frac{\delta}{2} \iint_R (4-x^2-y^2)^2\,dy\,dx$$

$$M_{xy} = \frac{\delta}{2}\int_0^{2\pi}\int_0^{2} (4 - r^2)^2\, r\, dr\, d\theta$$

$$\frac{\delta}{2}\int_0^{2\pi}\int_0^{2} (16 - 8r^2 + r^4)\, r\, dr\, d\theta$$

$$\frac{\delta}{2}\int_0^{2\pi}\int_0^{2} (16r - 8r^3 + r^5)\, dr\, d\theta$$

$$\frac{\delta}{2}\int_0^{2\pi} \left[8r^2 - 2r^4 + \frac{r^6}{6}\right]_0^2 d\theta$$

$$\frac{\delta}{2}\int_0^{2\pi} \left(32 - 32 + \frac{64}{6}\right) d\theta = 2\pi\left(10\tfrac{2}{3}\right) = \boxed{\frac{32\pi\delta}{3}}$$

$$M = 8\pi\delta \Rightarrow \bar{z} = 4/3$$

$$M = s^3 + \frac{2}{3}s^5 + \frac{1}{3}s^5 = s^3 + s^5,$$

$$M = s^3(1+s^2)$$

$$\Rightarrow \bar{x} = \frac{M_{yz}}{M} = \frac{s^4\left(\frac{1}{2} + \frac{7}{12}s^2\right)}{\cancel{s}^3(1+s^2)}$$

$$\boxed{\bar{x} = \frac{s\left(\frac{1}{2} + \frac{7}{12}s^2\right)}{1+s^2}}$$

$s=1:$ $\bar{x} = \dfrac{13/12}{2} = \dfrac{13}{24}$

$\bar{y} = 13/24$

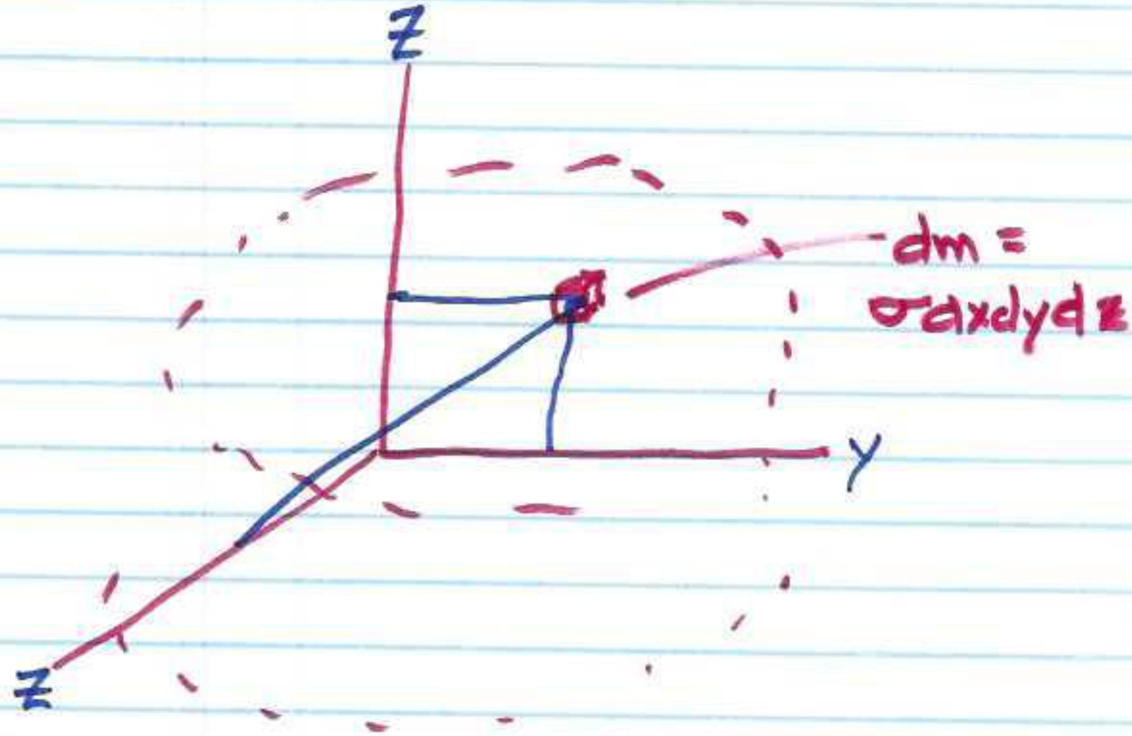
$\bar{z}$ $13/24$

torque $F \cdot d$

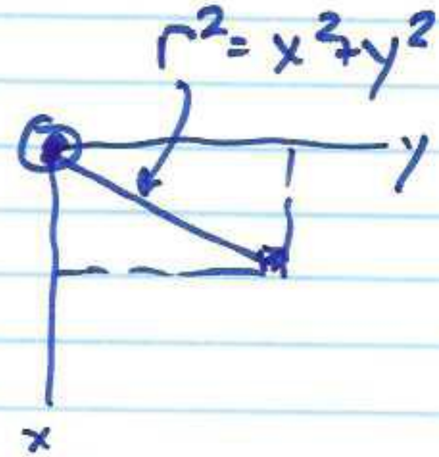
rotational accel

$F = ma$ $\qquad \tau = I\alpha$

$\frac{F}{a} = m$ $\qquad \frac{\tau}{\alpha} = I$



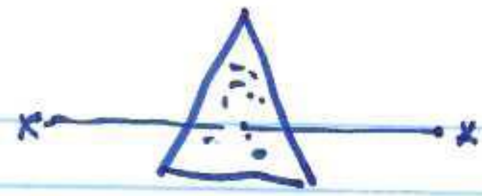
$dm = \sigma(x,y,z)\,dxdydz$

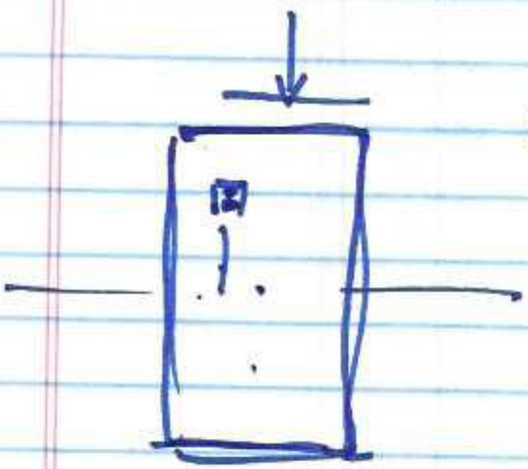


$dI_z = r^2 dm$

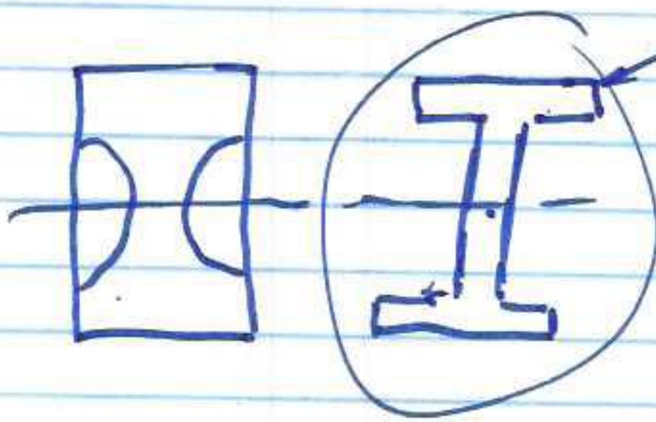
$dI_z = (x^2+y^2)\,dm =$

$$I_z = \iiint_{x\,y\,z} (x^2+y^2)\,\sigma(x,y,z)\,dxdydz$$

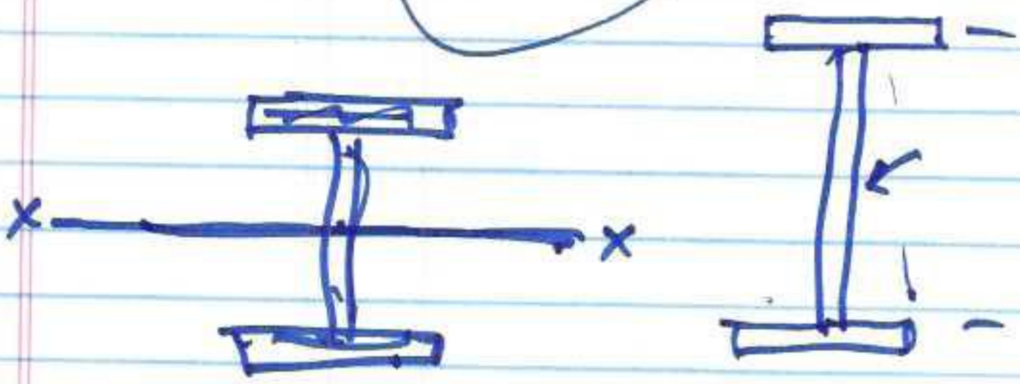




areal



most mass far from neutral axis



$$I_{xx} = \iint y^2 \delta \, dA$$