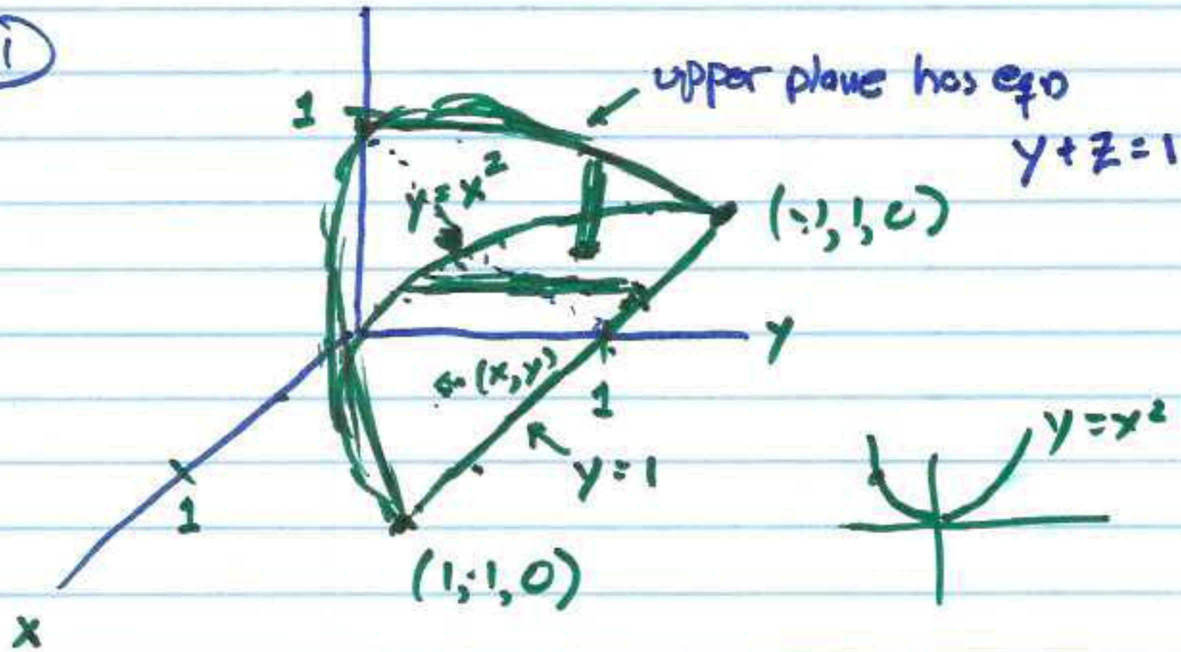
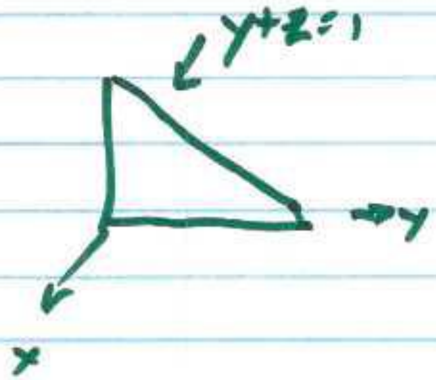
HEFC5

3/25

§15.5 Problems:

(21)





From base pt. (x,y) start @ 0, goes to $z=1-y$

2

$$V = \int_{-1}^{+1}\int_{x^2}^{1}\int_{0}^{1-y} (1)\, dz\, dy\, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} \Big[z\Big]_0^{1-y} dy\, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} (1-y)\, dy\, dx$$

$$= \int_{-1}^{1} \left[y - \frac{y^2}{2}\right]_{x^2}^{1} dx$$

$$= \int_{-1}^{1} \left[\left(1 - \frac{1}{2}\right) - \left(x^2 - \frac{x^4}{2}\right)\right] dx$$

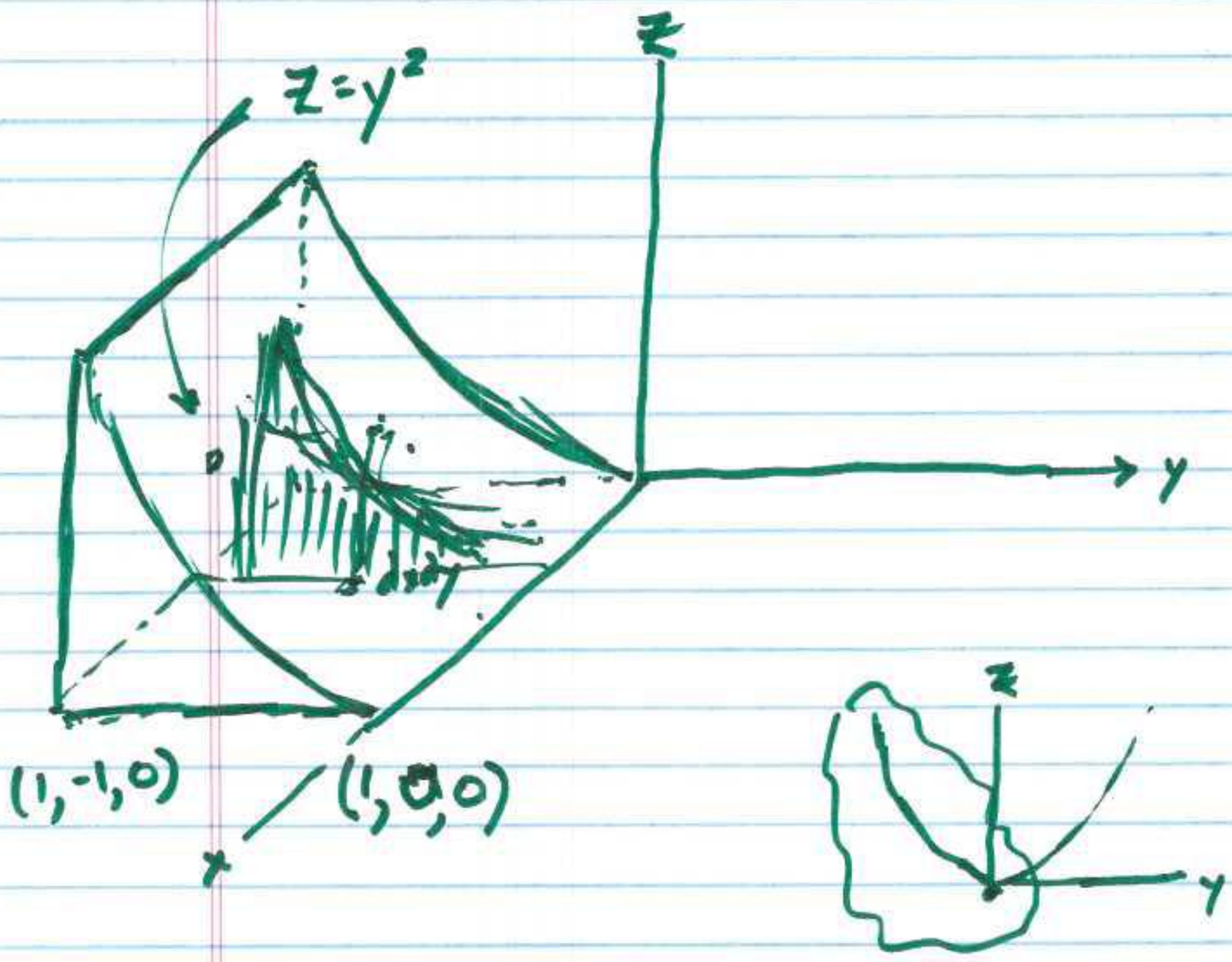
$$= \int_{-1}^{1} \left(\frac{1}{2} - x^2 + \frac{x^4}{2}\right) dx$$

3

$$= \left[\frac{x}{2} - \frac{x^3}{3} + \frac{x^5}{10} \right]_{-1}^{1}$$

$$= \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{10} \right) - \left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{10} \right) =$$

$$1 - \frac{2}{3} + \frac{2}{10} = \frac{30 - 20 + 6}{30} \sim \boxed{\frac{16}{30}}$$



$$V = \int_0^1 \int_{-1}^0 \int_0^{y^2} 1 \, dz\, dy\, dx$$

always for vol only

$$= \int_0^1 \int_{-1}^0 \left[z \right]_0^{y^2} dy\, dx \longrightarrow$$

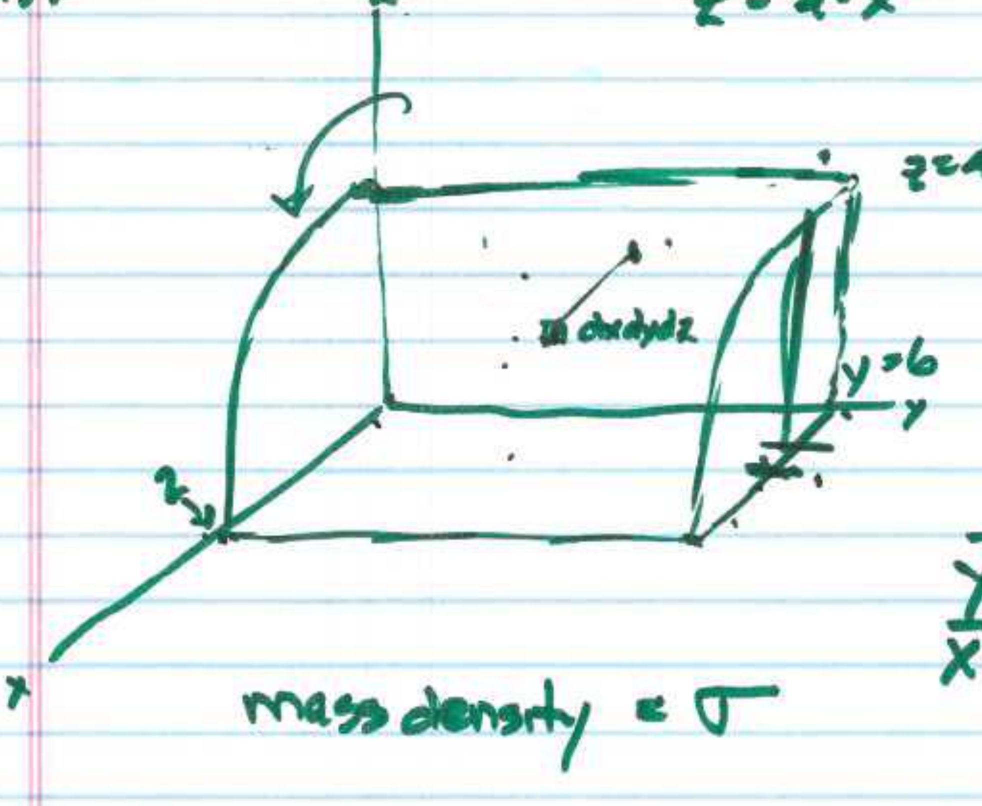
5

$$= \int_0^1 \int_{-1}^0 y^2 \, dy \, dx = \int_0^1 \left[\frac{y^3}{3}\right] dx =$$

$$\int \frac{1}{3} dx = \frac{x}{3}\Big|_0^1 = \underline{\frac{1}{3}}$$

Prob:

$z = 4 - x^2$



$\bar{y} = 3$
$\bar{x} = .75$

mass density $= \sigma$

$$M = \sigma \int_0^6 \int_0^2 \int_0^{4-x^2} dz\,dx\,dy$$

$$= \sigma \int_0^6 \int_0^2 \Big[z\Big]_0^{4-x^2} dx\,dy = \sigma \int_0^6 \int_0^2 (4-x^2)\,dx\,dy =$$

$$= \sigma \int_0^6 \left[4x - \frac{x^3}{3}\right]_0^2 dy = \sigma \int_0^6 \frac{16}{3}\,dy =$$

$$\sigma \left[\frac{16}{3} y\right]_0^6 = \sigma \cdot 32 = \boxed{32\sigma}$$

⑦ $\bar{x} = \dfrac{M_{yz}}{M}$

$$M_{yz} = \sigma \int_0^6 \int_0^2 \int_0^{4-x^2} x\,dz\,dx\,dy$$

$$= \sigma \int_0^6 \int_0^2 \int_0^{4-x^2} x\,dz\,dx\,dy = \sigma \int_0^6 \int_0^2 (4x - x^3)\,dx\,dy$$

$$= \sigma \int_0^6 \left[2x^2 - \frac{x^4}{4}\right]_0^2 dy =$$

$$\sigma \int_0^6 4\,dy = \sigma \left[4y\right]_0^6 = \underline{24\sigma}$$

So... $\bar{x} = \dfrac{24\sigma}{32\sigma} = \underline{0.75}$