WSNJ4

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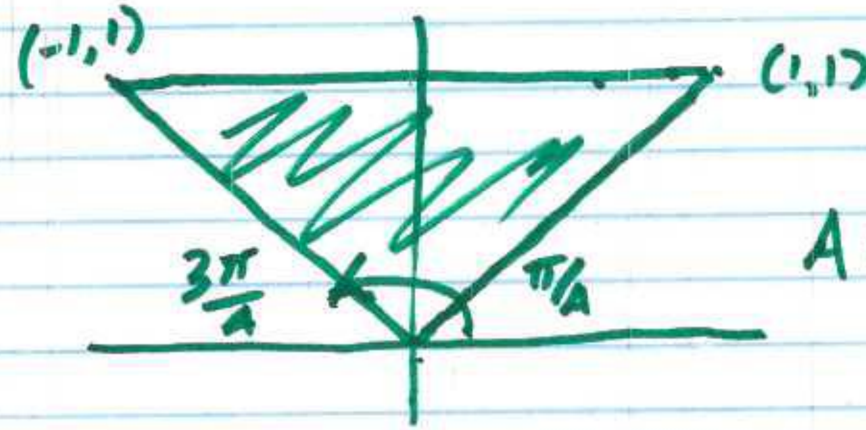
3/23

Old Business:



$A = 1\cdot 2/2 = 1$

$r\sin\theta = 1$

or $r = \dfrac{1}{\sin\theta}$

$$A = \int_{\pi/4}^{3\pi/4}\int_0^{\frac{1}{\sin\theta}} r\,dr\,d\theta = \int_{\pi/4}^{3\pi/4}\left[\frac{r^2}{2}\right]_0^{1/\sin\theta} d\theta$$

$$= \frac{1}{2}\int_{\pi/4}^{3\pi/4}\frac{d\theta}{\sin^2\theta} = -\frac{1}{2}\cot\Big|_{\pi/4}^{3\pi/4}$$

$$-\frac{1}{2}\left(\cot\frac{3\pi}{4}-\cot\frac{\pi}{4}\right) = ?\ 1$$

Problem:

$$V = \iiint \underbrace{dx\,dy\,dz}_{\text{differential cartesian box}}$$

$$V = \int_{-2}^{2} \int_{-\sqrt{\frac{4-x^2}{2}}}^{+\sqrt{\frac{4-x^2}{2}}} \int_{x^2+3y^2}^{8-x^2-y^2} dz\,dy\,dx$$

$$V = \int_{-2}^{2} \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} \Big[z \Big]_{x^2+3y^2}^{8-x^2-y^2} dy\,dx$$

$$V = \int_{-2}^{2} \int_{-\sqrt{\frac{4-x^2}{2}}}^{+\sqrt{\frac{4-x^2}{2}}} \left[(8-x^2-y^2)-(x^2+3y^2)\right] dy\,dx$$

$$= \int_{-2}^{2} \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} (8-2x^2-4y^2)\,dy\,dx$$

$$= \int_{-2}^{2} \left[8y-2x^2y-\frac{4y^3}{3}\right]_{-\sqrt{\frac{4-x^2}{2}}}^{+\sqrt{\frac{4-x^2}{2}}} dx$$

④

$$\int_{-2}^{2}\left[\left(8\sqrt{\frac{4-x^2}{2}}-2x^2\sqrt{\frac{4-x^2}{2}}-\frac{4}{3}\left(\frac{4-x^2}{2}\right)^{3/2}\right)-\right.$$

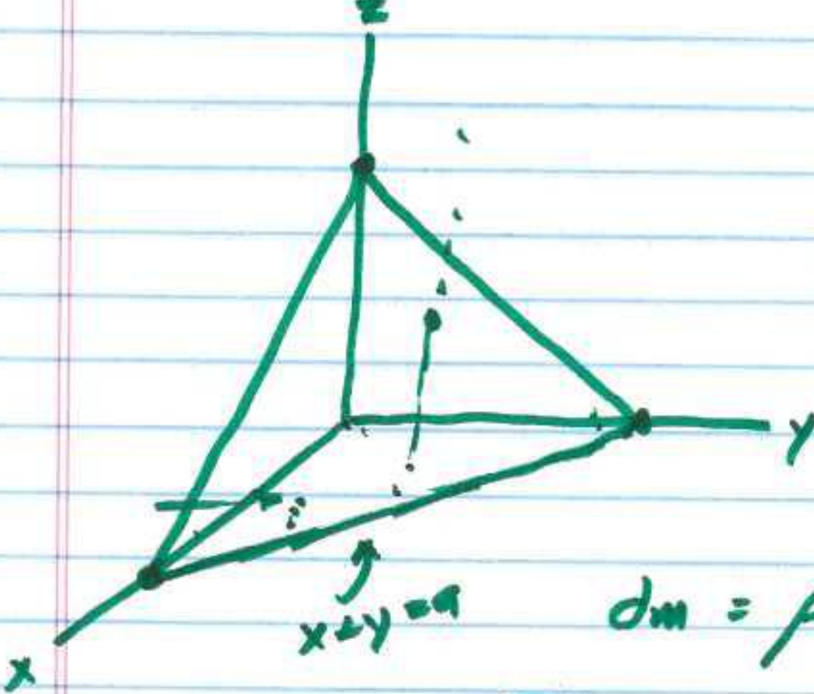
$$\left.\left(8\left(-\sqrt{\frac{4-x^2}{2}}\right)+2x^2\sqrt{\frac{4-x^2}{2}}-\frac{4}{3}\left(\frac{4-x^2}{2}\right)^{3/2}\right)\right]dx$$

$$=\int_{-2}^{2}\left(16\sqrt{\frac{4-x^2}{2}}-4x^2\sqrt{\frac{4-x^2}{2}}-\frac{8}{3}\left(\frac{4-x^2}{2}\right)^{3/3}\right)dx$$

$$=\int_{-2}^{2}(16-4x^2)\sqrt{\frac{4-x^2}{2}}-\frac{8}{3}\left(\frac{4-x^2}{2}\right)^{3/2}dx$$

$x = 2\sin u$ yields

$$=\frac{4\sqrt{2}}{3}\int_{-2}^{2}(4-x^2)^{3/2}\,dx = 8\pi\sqrt{2}$$



$x+y+z=a$

$dm = \rho(x,y,z)\,dz\,dy\,dx$

1

$$V = \iiint dz\,dy\,dx$$

$$V = \int_0^a \int_0^{a-x} \int_0^{a-x-y} dz\,dy\,dx$$

DO!

5.5

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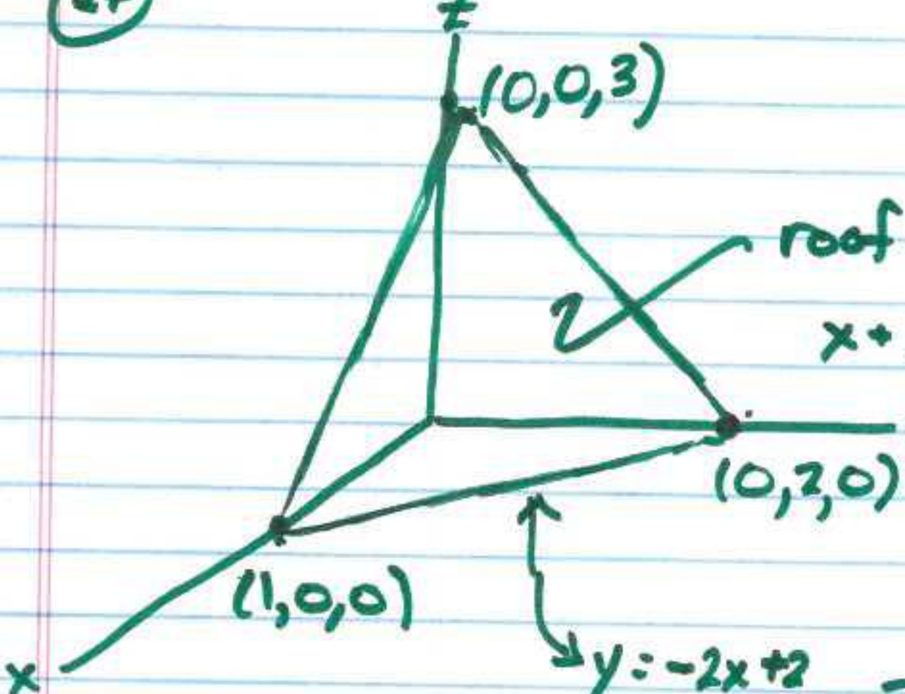

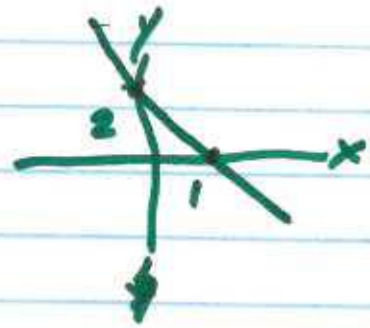

$ax + by + cz = d$

$3c = d$

$2b = d$

$a = d$

$\frac{z}{3} = 1 - x - \frac{y}{2}$

$z = 3 - 3x - \frac{3}{2}y$

$dx + \frac{d}{2}y + \frac{d}{3}z = d$

$x + \frac{1}{2}y + \frac{1}{3}z = 1$ ✓

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$$V = \int_0^a \int_0^{a-x} \left[z \right]_0^{a-x-y} dy\,dx$$

$$= \int_0^a \int_0^{a-x} (a-x-y)\,dy\,dx$$

$$= \int_0^a \left[ay - xy - \frac{y^2}{2} \right]_0^{a-x} dx$$

$$= \int_0^a \left(\cancel{(a^2 - xy) - (a-x)(xy)} - \frac{1}{2}(a-x)^2 \right) dx$$

$$= \int_0^a \cancel{(a-x)\left[ay - xy - \frac{1}{2}(a-x)\right.}$$

$$\int_0^a a(a-x) - x(a-x) - \frac{1}{2}(a-x)^2\,dx$$

$$= \int_0^a \left(a^2 \underbrace{- ax - ax}_{-2ax} + x^2 - \frac{1}{2}\left(a^2 - 2ax + x^2\right)\right) dx$$

$$\frac{a^2}{2} \quad -2ax + \frac{x^2}{2}$$

$$-ax$$

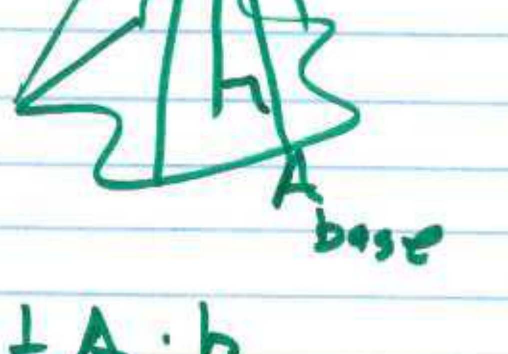
$$= \int_0^a \left(\frac{a^2}{2} - ax + \frac{x^2}{2}\right) dx$$

$$= \left[\underbrace{\frac{a^2x}{2} - \frac{ax^2}{2}}_{0} + \frac{x^3}{6}\right]_0^a$$

$$= \frac{a^3}{6}$$

Base $= \frac{a^2}{2}$

ht $= a$



$$\frac{1}{3}\left(\frac{a^2}{2}\right)a = \frac{a^3}{6}$$

$$V = \frac{1}{3} A_{base} \cdot h$$

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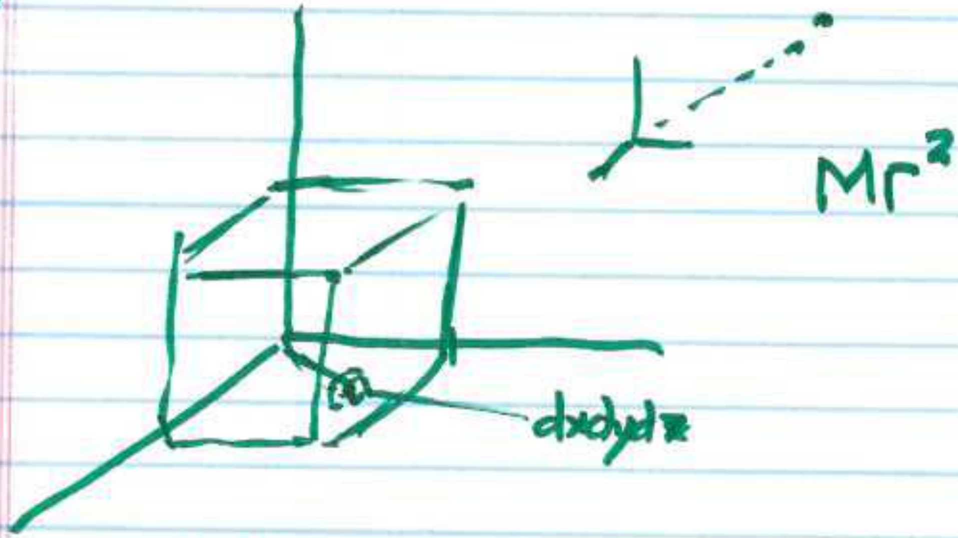

$$\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)\,dx\,dy\,dz$$

$$\int_0^1\int_0^1 \left[\frac{x^3}{3}+xy^2+xz^2\right]_0^1 dy\,dz$$

$$\int_0^1\int_0^1 \left(\frac{1}{3}+y^2+z^2\right) dy\,dz$$

$$\int_0^1 \left[\frac{y}{3}+\frac{y^3}{3}+yz^2\right]_0^1 dz = \int_0^1 \left(\frac{1}{3}+\frac{1}{3}+z^2\right) dz$$

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$$\left[\frac{2}{3}z+\frac{z^3}{3}\right]_0^1=\frac{2}{3}+\frac{1}{3}=1$$