MU68K

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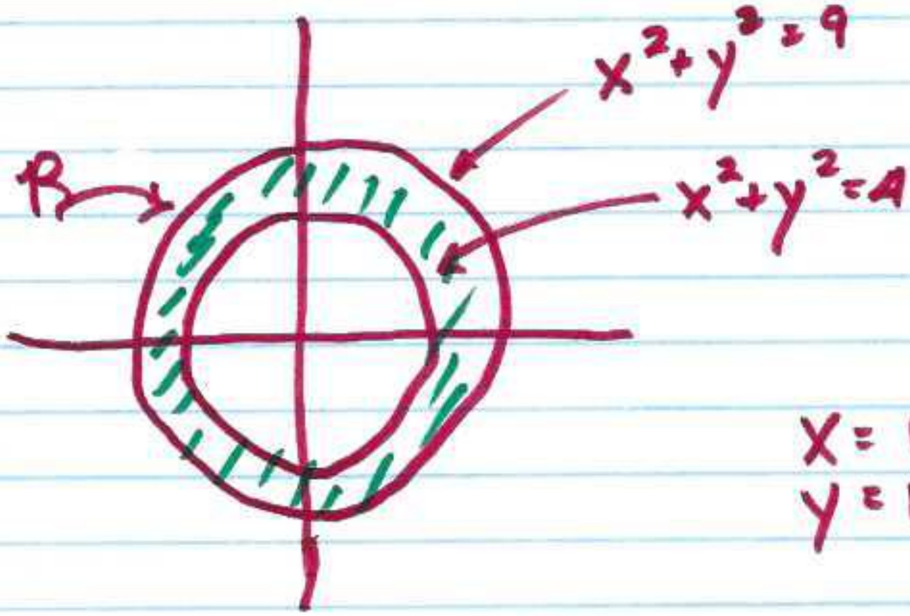
$$\iint_R f(x,y)\,dx\,dy = \iint_{R'} f(r\cos\theta, r\sin\theta)\,\underline{r}\,dr\,d\theta$$

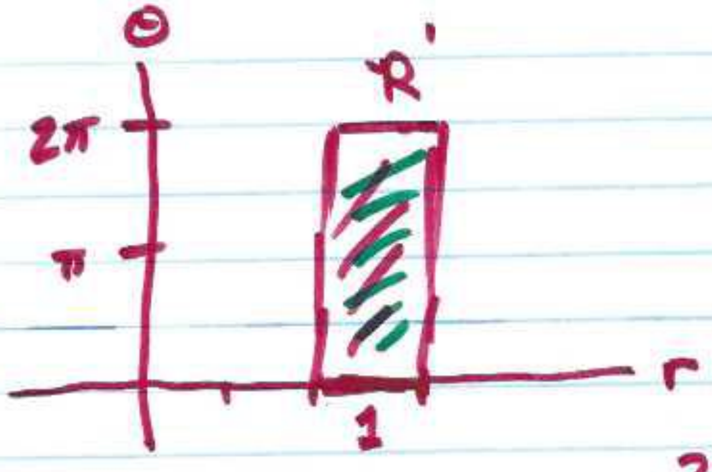


$x = r\cos\theta$
$y = r\sin\theta$

Eval: $\iint_R \sqrt{x^2+y^2}\,dx\,dy$ X-form

$$\iint_{R'} \underbrace{\sqrt{r^2\cos^2\theta + r^2\sin^2\theta}}\, r\,dr\,d\theta$$

②



$$\int_0^{2\pi}\int_2^3 r^2\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^3}{3}\right]_2^3 d\theta =$$

$$\frac{19}{3}\int_0^{2\pi} d\theta = \frac{38}{3}\pi$$ area between circles

not same

$9\pi - 4\pi = 5\pi$

Lemniscate

$$r^2 = a^2 \cos 2\theta$$

$$r = a\sqrt{\cos 2\theta}$$

$r\,dr\,d\theta$

$$\text{Area} = 4\int_0^{\pi/4}\int_0^{a\sqrt{\cos 2\theta}} r\,dr\,d\theta$$

$$= 4\int_0^{\pi/4}\int_0^{a\sqrt{\cos 2\theta}} r\,dr\,d\theta$$

$$= 4\int_0^{\pi/4}\left[\frac{r^2}{2}\right]_0^{a\sqrt{\cos 2\theta}} d\theta = \frac{4}{2}\int_0^{\pi/4} a^2 \cos 2\theta\, d\theta$$

④ $\boxed{\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}}$

$$A = 3a^2\int_0^{\pi/4}\cos 2\theta\, d\theta = 2a^2\left[\frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= a^2(1) = \underline{\underline{a^2}}$$

$$\int_{\pi/4}^{3\pi/4}\int_0^{\sqrt{2}\cdot\sin\theta} r\,dr\,d\theta = A \qquad \boxed{\sin^2\theta =}$$

$$A = \int_{\pi/4}^{\frac{3\pi}{4}}\left[\frac{r^2}{2}\right]_0^{\sqrt{2}\sin\theta} d\theta = \int_{\pi/4}^{3\pi/4}\sin^2\theta\, d\theta = \int_{\frac{\pi}{4}}^{3\pi/4}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

(5)

$$\int_{\frac{\pi}{4}}^{3\pi/4} \frac{d\theta}{2} - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\cos 2\theta}{2}\, d\theta$$

$$\downarrow$$

$$\frac{1}{2}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) - \frac{1}{4}\Big[\sin 2\theta\Big]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$\frac{\pi}{4} - \frac{1}{4}\Big[(-1) - 1\Big]$$

$$\frac{\pi}{4} + \frac{1}{2}$$

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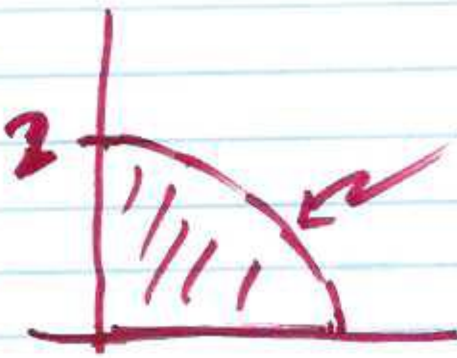
$$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2+y^2)\,dx\,dy$$



$x^2+y^2=4$

$\Rightarrow x = \sqrt{4-y^2}$

$$\int_0^{\pi/2} \int_0^2 (r^2)(r)\,dr\,d\theta = ?$$

$$\int_0^{\pi/2} \left[\frac{r^4}{4}\right]_0^2 d\theta = \frac{16}{4}\int_0^{\pi/2} d\theta = 4\cdot\frac{\pi}{2} = 2\pi$$

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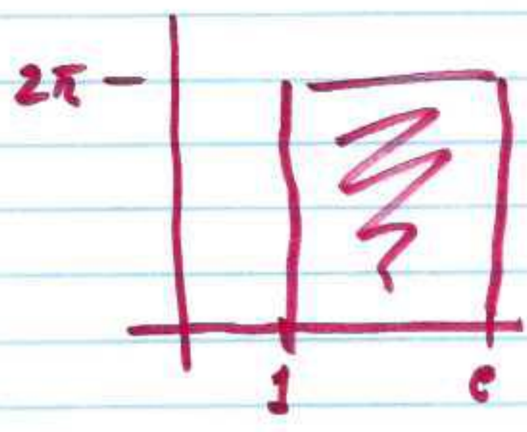
$\iint$ $\qquad R: 1 \le x^2+y^2 \le e$

$$\iint_R \frac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}\,dx\,dy$$



$$\int_0^{2\pi}\int_1^{e} \frac{\ln r^2}{\cancel{\sqrt{r}}}\,\cancel{r}\,dr\,d\theta$$

$$\int_0^{2\pi}\int_1^{e} 2\ln r\,dr\,d\theta =$$

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Note: $x(\ln x - 1)$

$$2\int_0^{2\pi}\int_1^{e} \ln r\, dr\, d\theta = 2\int_1^{2\pi}\Big[r\ln r - r\Big]_1^{e} d\theta$$

$$\underbrace{\Big[(e(1)-e) - (0)\Big]}_{0} + 1$$

$$2\int_0^{2\pi} d\theta = 4\pi$$

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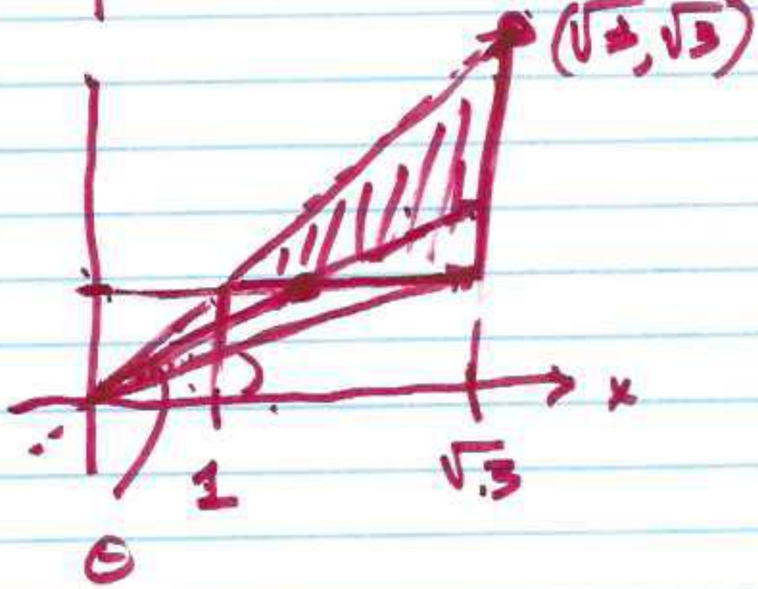
15 $$\int_1^{\sqrt{3}}\int_{\underbrace{1}}^{x} dy\, dx = \int_1^{\sqrt{3}} (x-1)\,dx = \left[\frac{x^2}{2} - x\right]_1^{\sqrt{3}}$$

$$\left(\frac{3}{2} - \sqrt{3}\right) - \left(\frac{1}{2} - 1\right) = \_\_\_\_$$

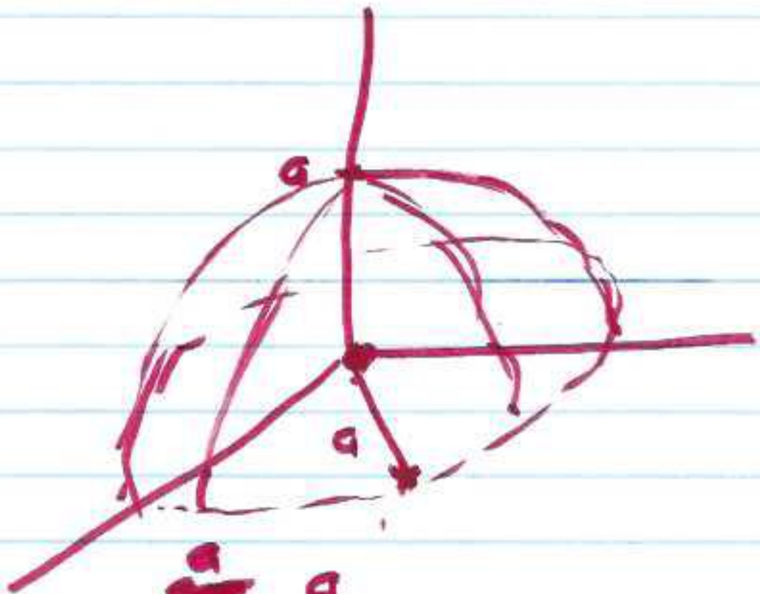
⑨



What is average height of hemisphere?

$$x^2+y^2+z^2=a^2$$

$$z=+\sqrt{a^2-x^2-y^2}$$



$$\bar{z} = \frac{A}{\pi a^2} \int_0^a \int_0^a \sqrt{a^2 - x^2 - y^2}\, dx\,dy$$

→ polor

$$\bar{z} = \frac{1}{\pi a^2} \int_0^{2\pi} \int_0^a \sqrt{a^2 - r^2}\; r\,dr\,d\theta$$

$$u = a^2 - r^2$$

$$du = -2r\,dr$$