SBX8U ①

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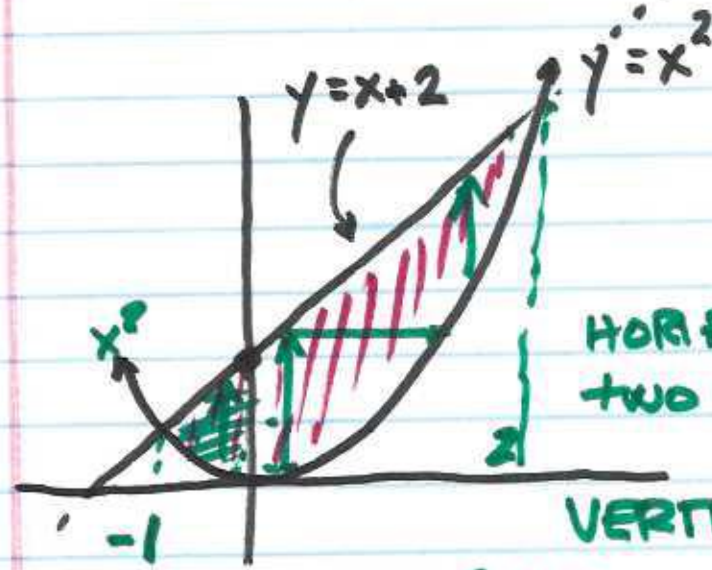
Area = $\iint f(x,y)\,dx\,dy$ ...

Ex:



$x^2 = x+2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

HORIZONTAL requires two terms

VERTICAL

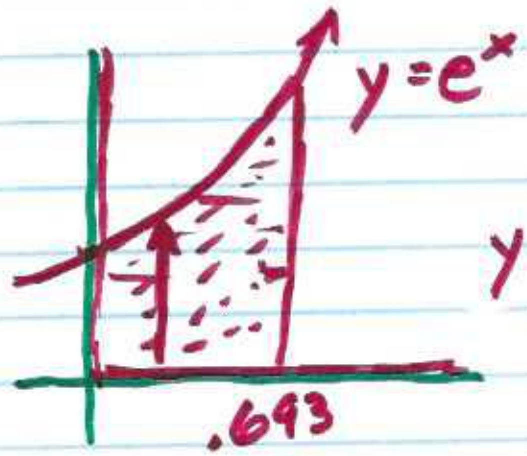
$$A = \int_{-1}^{2}\int_{x^2}^{x+2} \underline{1} \; dy\,dx$$

$$= \int_{-1}^{2} \Big[y\Big]_{x^2}^{x+2} dx$$

$$= \int_{-1}^{2} (x+2-x^2)\,dx$$

$$A = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} = \ldots ?$$

5. $y = e^x$, lines $y=0$, $x=0$, $x=\ln 2$



y first

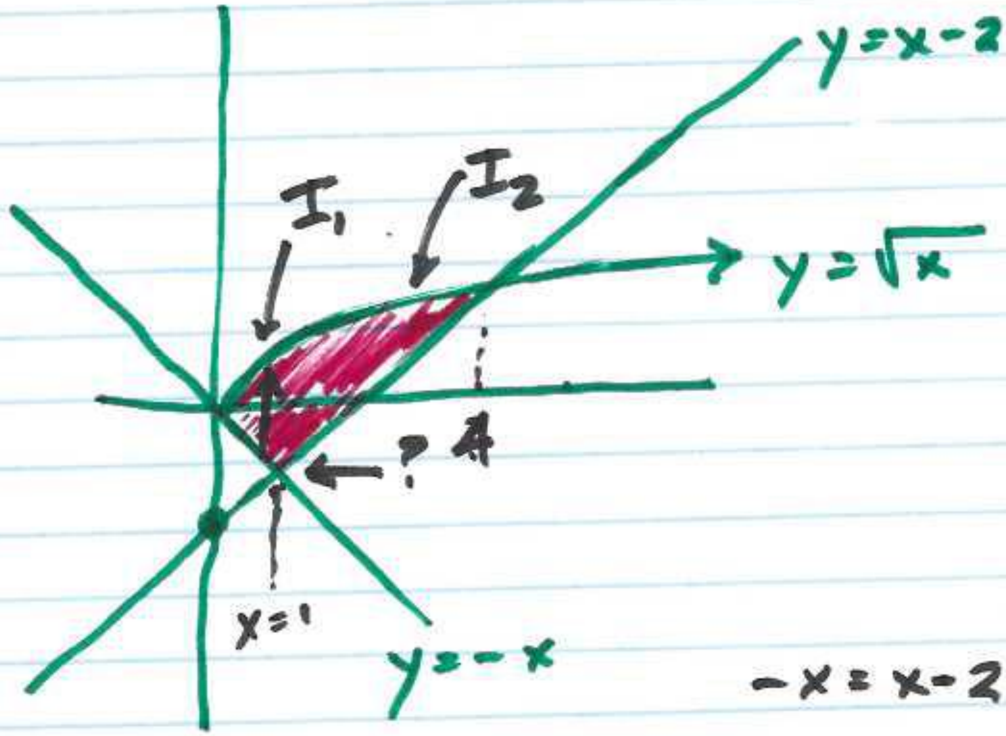
$$A = \int_0^{\ln 2}\int_0^{e^x} 1\, dy\, dx$$

$$= \int_0^{\ln 2}[y]_0^{e^x}dx = \int_0^{\ln 2} e^x\, dx = [e^x]_0^{\ln 2} = 2$$

$2 - 1 = 1$

(12) Area between $y = x-2$, $y = -x$ &

$y = \sqrt{x}$



$-x = x-2$

$0 = 2x-2$

or $x = 1$

Piece #1 $x \in [0,1]$

#2 $x \in [1,4]$

(4)

$$I_1 = \int_0^1 \int_{-x}^{\sqrt{x}} dy\,dx = \int_0^1 [y]_{-x}^{\sqrt{x}}\,dx$$

$$I_2 = \int_1^4 \int_{x-2}^{\sqrt{x}} dy\,dx =$$

$$I_1 = \int_0^1 (\sqrt{x} + x)\,dx = \left(\frac{2}{3}x^{3/2} + \frac{x^2}{2}\right)\Big|_0^1 = 7/6$$

$$I_2 = \int_1^4 (\sqrt{x} - x + 2)\,dx = 2$$

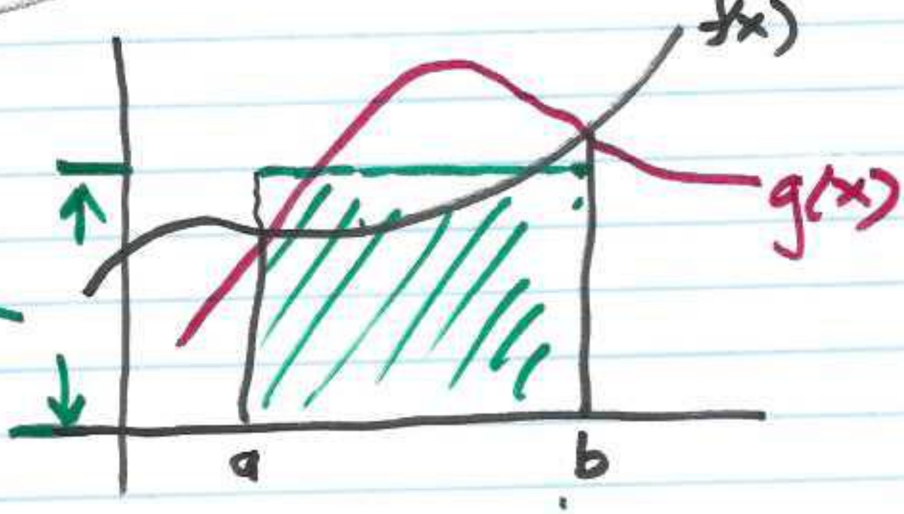
$$\left(\frac{2}{3}x^{3/2} - \frac{x^2}{2} + 2x\right)\Big|_1^4 = 2$$

$$\left(\frac{16}{3} - \underline{8 + 8}\right) - \left(\frac{2}{3} - \frac{1}{2} + 2\right) = \_\_\_$$

$$\frac{14}{3} + \frac{3}{2} = 37/6$$

(5)

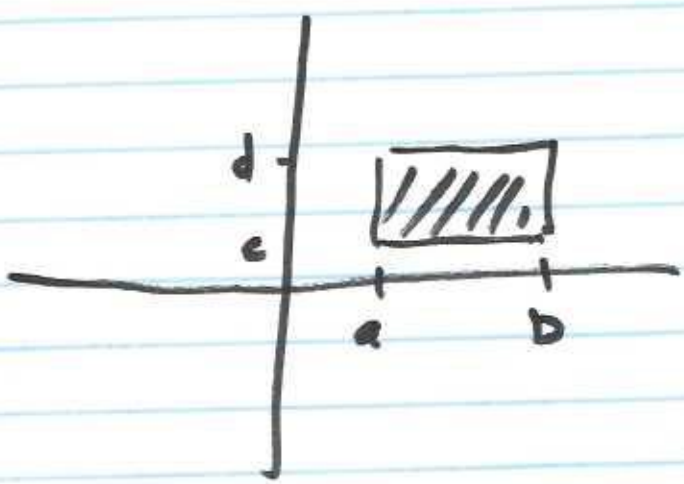
$$\bar{f}_{[a,b]} = \int_a^b f(x)\,dx \cdot \left(\frac{1}{b-a}\right)$$



$$\frac{f(a)+f(b)}{2} = \text{gross avg}$$

$$R = [a,b] \times [c,d]$$

$$\bar{f}_{[R]}(x,y)$$



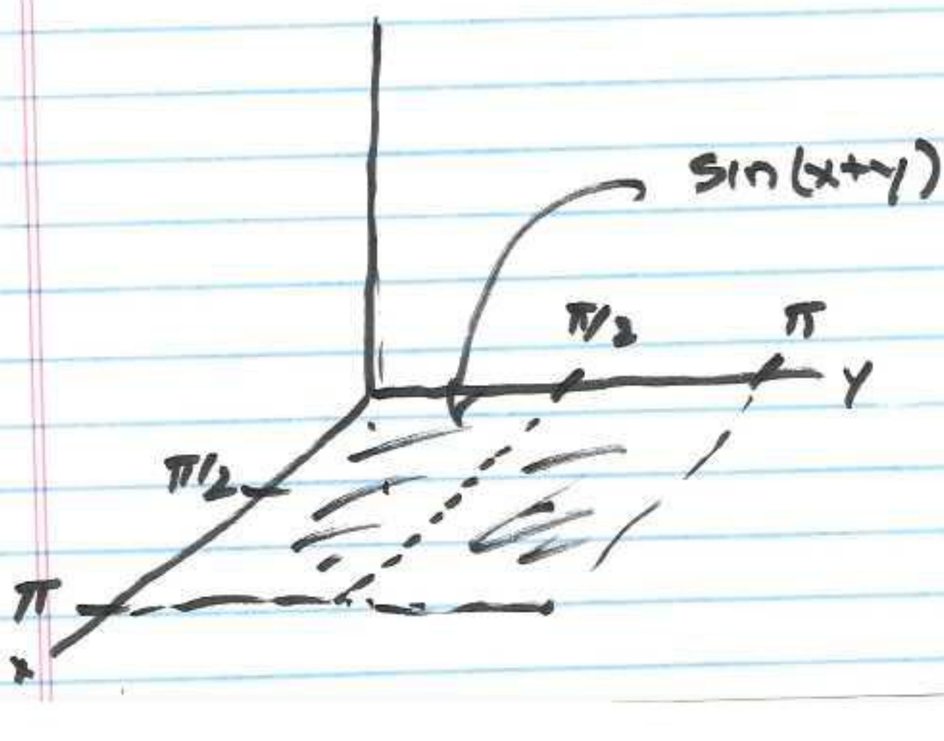
$$\frac{1}{(d-c)(b-a)}\int_a^b\int_c^d f(x,y)\,dy\,dx = \bar{f}_R(x,y)$$

(19) Find the average value of $\sin(x+y)$

over

(a) $0 \le x \le \pi,\ 0 \le y \le \pi$ Done $\underline{\underline{=0}}$

(b) $0 \le x \le \pi,\ 0 \le y \le \pi/2$ - ?



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$$\frac{1}{\pi^2/2}\int_0^{\pi}\underbrace{\int_0^{\pi/2}\sin(x+y)\,dy}\,dx$$

$$\left[-\cos(x+y)\right]_0^{\pi/2} =$$

$$-\cos\left(x+\tfrac{\pi}{2}\right)+\cos(x)$$

$$\bar{f} = \frac{2}{\pi^2}\int_0^{\pi}\left[\cos x - \underbrace{\cos\left(x+\tfrac{\pi}{2}\right)}\right]dx$$

$$\cos x \underbrace{\cos\tfrac{\pi}{2}}_{0} - \sin x(1)$$

so avg. value is $\dfrac{4}{\pi^2}$

$$\frac{2}{\pi^2}\int_0^{\pi}(\cos x+\sin x)\,dx$$

$$= \frac{2}{\pi^2}\left[\sin x - \cos x\right]_0^{\pi} = \frac{2}{\pi^2}(1+1) = \frac{4}{\pi^2}$$

254 counties in TX

@ t_0, each station records temp

Let $T_i(t)$ = temp @ time t of county i. Let A_i = area of county i.

$$\bar{T} = \frac{1}{\sum_{i=1}^{254} A_i} \sum_{i=1}^{254} A_i T_i(t_0)$$

$$\frac{1}{\iint dx\,dy} \iint E(x,y)\,dx\,dy$$ avg elev

Recall from 1-D calc.

$$\boxed{\frac{dx}{dt}dt = dy}$$

$$\int f(x)\,dx = \int f(x(t))\,\underbrace{x'(t)\,dt}_{dx} \qquad x = x(t)$$

In 2-D $\quad x = x(u,v),\ y = y(u,v)$

$$\iint f(x,y)\,dx\,dy$$

adj factor

$$\iint f\big(x(u,v), y(u,v)\big)\ \underline{\qquad}\ du\,dv$$

adjustment factor is called the "jacobian" determinant

$$\begin{array}{l} \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial y}{\partial u} \\[2ex] \dfrac{\partial x}{\partial v} & \dfrac{\partial y}{\partial v} \end{vmatrix} \end{array} \begin{array}{l} \leftarrow \text{old} \\ \leftarrow \text{new} \\ \leftarrow \text{old} \\ \leftarrow \text{new} \end{array} = \frac{\partial(x,y)}{\partial(u,v)}$$

$$\iint f\big(x(u,v), y(u,v)\big)\ \underbrace{\frac{\partial(x,y)}{\partial(u,v)}\,du\,dv}_{dx\,dy}$$

Let $x = x(r,\theta) \Rightarrow x = r\cos\theta$

$y = y(r,\theta) \Rightarrow y = r\sin\theta$

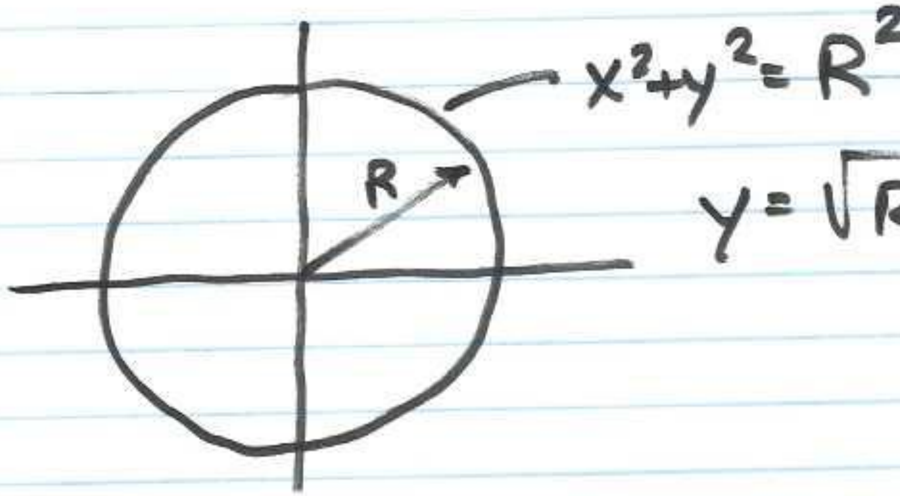
$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix} =$$

$$r\cos^2\theta - (-r\sin^2\theta) = r(\cos^2\theta + \sin^2\theta)$$

$$= r(1) = r$$

$$dx\,dy = r\,dr\,d\theta$$

Area of Circle:



$y=\sqrt{R^2-x^2}$

$x=r\cos\theta \quad y=r\sin\theta$

$$\int_0^{2\pi}\int_0^{R} r\,dr\,d\theta \;(dx\,dy) = \int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^{R} d\theta =$$

$$\frac{R^2}{2}\int_0^{2\pi} d\theta = \frac{R^2}{2}\left[\theta\right]_0^{2\pi} = \frac{R^2}{\cancel{2}}\cancel{2}\pi$$

πR^2