QGG4L

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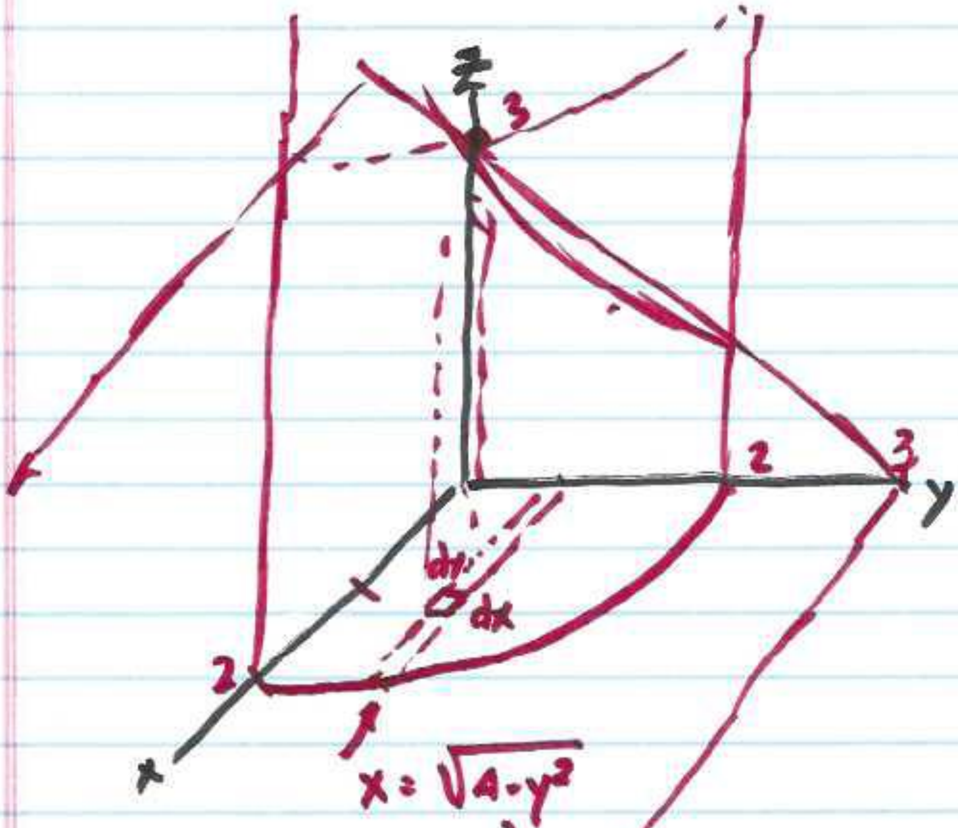
60) Bounding surfaces:

xy plane is floor

$z = 3-y$ is roof

the part of cylinder $x^2+y^2=4$ is the (curved) front wall



$$\text{Vol} = \int_0^2 \int (3-y)\,dx\,dy$$

2

$$V = \int_0^2 \left[3x - xy\right]_0^{\sqrt{4-y^2}} dy$$

$$\int_0^2 \left(3\sqrt{4-y^2} - y\sqrt{4-y^2}\right) dy$$

$$3\int_0^2 \sqrt{4-y^2}\, dy$$

$$y = 2u \quad dy = 2du$$

$$3\int_{u=0}^{1} 2\sqrt{1-u^2}\,(2du) = 12\int_0^1 \sqrt{1-u^2}\, du$$

$$12\left[\frac{u}{2}\sqrt{1-u^2} + \frac{1}{2}\arctan\frac{u}{\sqrt{1-u^2}}\right]_0^1$$

③

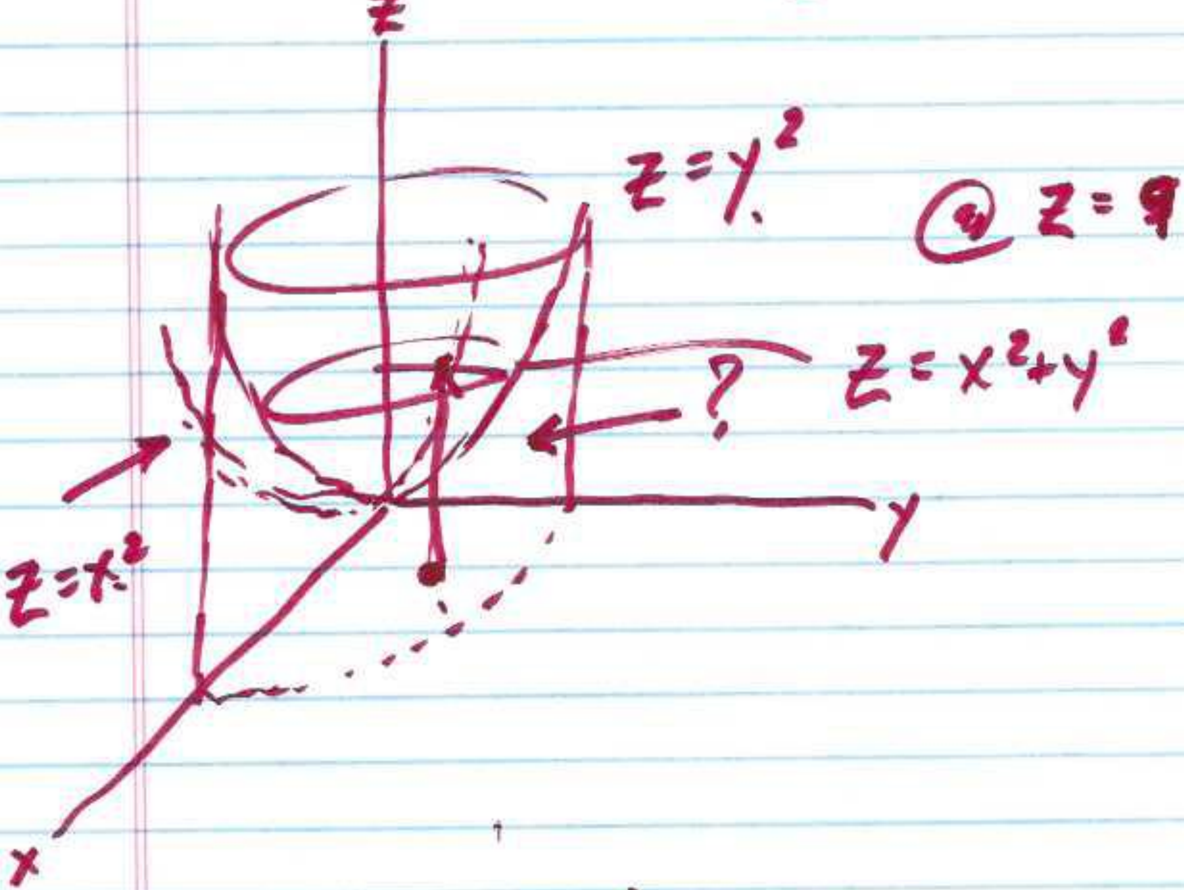
Start Over:

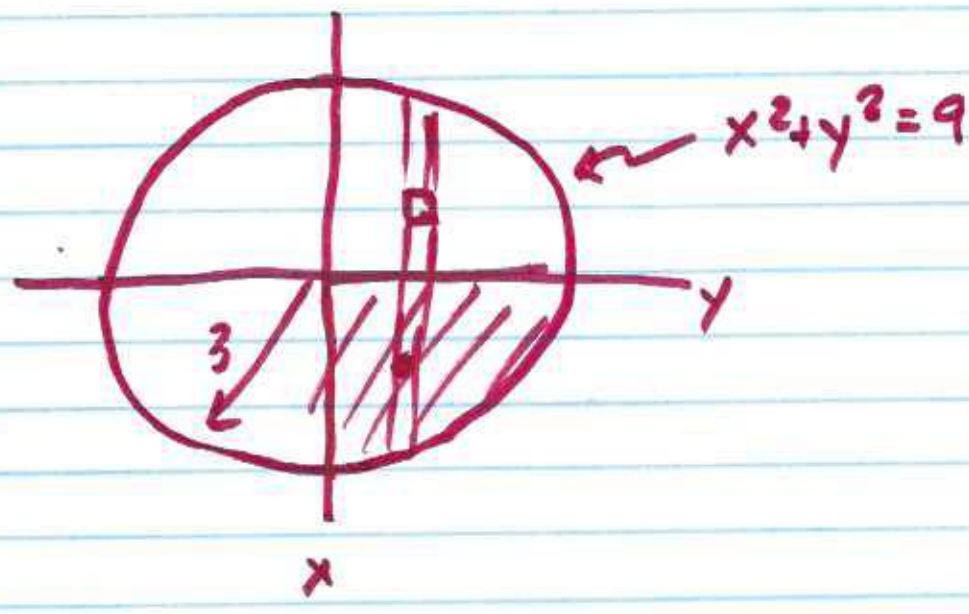
$$V = \int_0^2 \int_0^{\sqrt{4-x^2}} (3-y) \quad dy\,dx$$

$$= \int_0^2 \left[3y - y^2\right]_0^{\sqrt{4-x^2}} \quad dx$$

$$= \int_0^2 \left(3\sqrt{4-x^2} - (4-x^2)\right) dx$$

$$\text{Vol} = \left[3\int_0^2 \sqrt{4-x^2}\right] - \left[4x - \frac{x^3}{3}\right]_0^2$$


z
$z = y^2$
@ $z = 9$
$z = x^2 + y^2$
?
$z = x^2$
y
x


$x^2 + y^2 = 9$
3
y
x

5

$$\text{Vol} = 4\int_0^3\int_0^{\sqrt{9-y^2}} x^2+y^2 \, dx\,dy$$

$$= 4\int_0^3 \left[\frac{x^3}{3} + xy^2\right]_0^{\sqrt{9-y^2}} dy$$

$$= 4\int_0^3 \left(\frac{1}{3}(9-y^2)^{3/2} + y^2(9-y^2)^{1/2}\right) dy$$

$$= \boxed{127.2}$$

$x \to r\cos\theta$

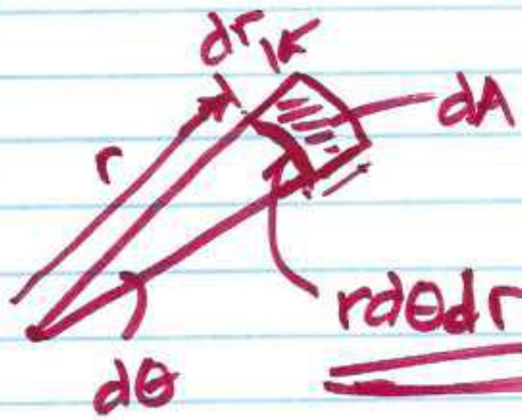
$y \to r\sin\theta$

$z \to z$

$x = r\cos\theta$
$y = r\sin\theta$

Jacobian

$$\iint_{xy} f(x,y)\,dx\,dy \rightsquigarrow \iint_{r\theta} f(x(r,\theta), y(r,\theta))\, r\,dr\,d\theta$$



$$\underline{r\,d\theta\,dr = dA}$$

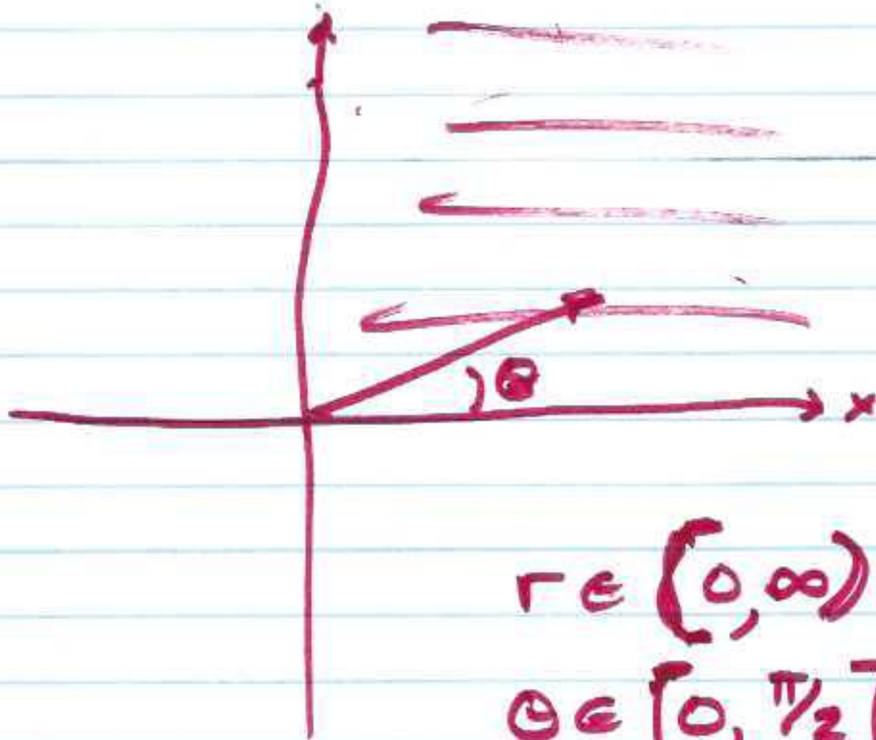
$$I = \int_0^\infty \underline{e^{-x^2}}\,dx \qquad \int_0^\infty x e^{-x^2}\,dy$$

$$I^2 = \int_0^\infty e^{-x^2}\,dx \cdot \int_0^\infty e^{-y^2}\,dy$$

$$= \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$$

⑦

$r^2 = x^2 + y^2$



$r \in [0, \infty)$

$\theta \in [0, \pi/2]$

$$I^2 = \int_0^{\pi/2}\int_0^{\infty} e^{-r^2} r \cdot \, dr\, d\theta$$

Look @ $\displaystyle\int_0^{\infty} \underbrace{r e^{-r^2} dr} = ?$

Let $u = r^2$ $du = 2r\,dr$

$$\int_{\theta=0}^{\pi/2}\int_{u=0}^{\infty} e^{-u}\left(\frac{du}{2}\right)d\theta$$

$$\int_{0}^{\pi/2}\left[\frac{-e^{-u}}{2}\right]_{0}^{\infty} d\theta$$

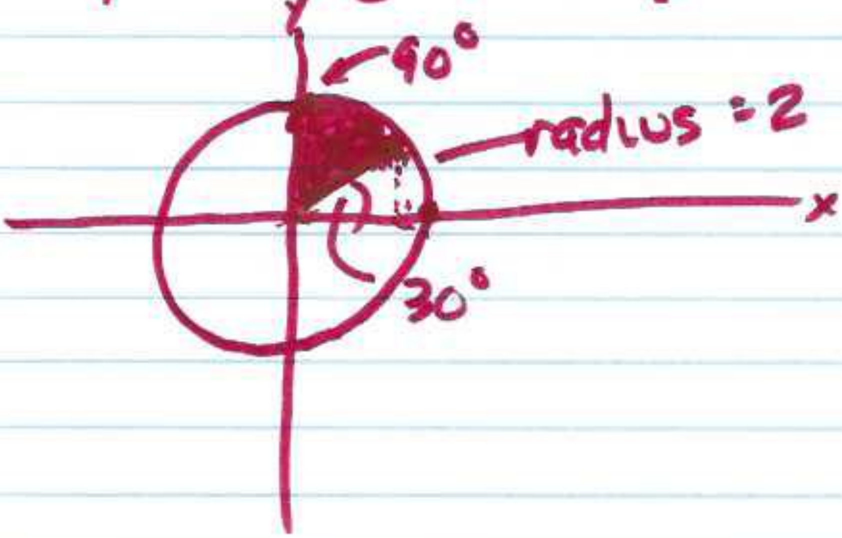
$$\frac{1}{2}$$

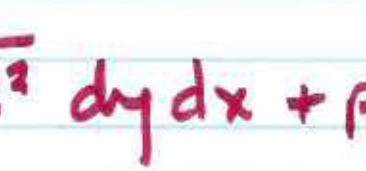
$$= \int_{0}^{\pi/2}\left(\frac{1}{2}\right)d\theta = \left[\frac{\theta}{2}\right]_{0}^{\pi/2} = \frac{\pi}{4} = I^{2}$$

$$I = \int_{0}^{\infty} e^{-x^{2}}dx = \frac{\sqrt{\pi}}{2}$$

$$n(0;1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^{2}}{2}}$$

Integrate: $f(x,y) = \sqrt{4-x^2}$ over the sector cut from the disk $x^2+y^2 \leq 4$ by the rays $\theta = \frac{\pi}{6}$ & $\theta = \frac{\pi}{2}$





$$\text{Vol} = \int_0^{\sqrt{3}} \int_0^{2} \sqrt{4-x^2}\, dy\, dx + \text{part @ top}$$

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$y = \sqrt{4-x^2}$