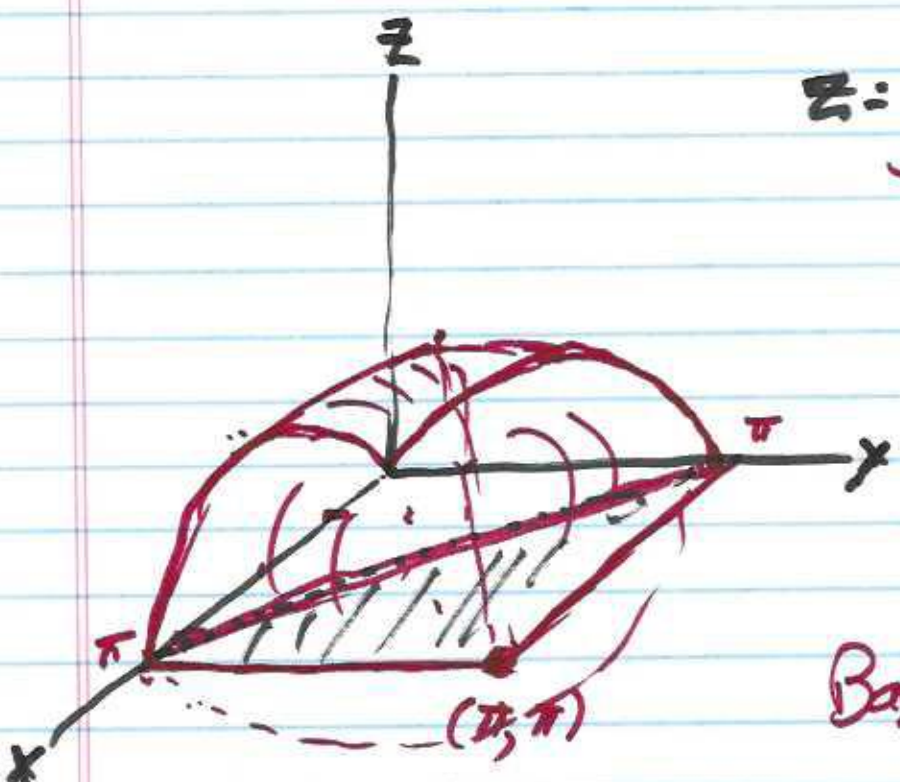
WN6X4

3/11

①

What is average value of $\sin(x+y)$ over the square $[0,\pi]\times[0,\pi]$?



$z = \sin(x+y)$

Base Area $= \pi^2$

$$\overline{\sin(x+y)} = \frac{1}{\pi^2}\int_0^\pi\int_0^\pi \sin(x+y)\,dx\,dy$$

(2)

$$\frac{1}{\pi^2}\int_0^{\pi}\left(\underbrace{\int_0^{\pi}\sin(x+y)\,dx}_{\left[-\cos(x+y)\right]_0^{\pi}}\right)dy$$

$$\left[-\cos(\pi+y)+\cos(0+y)\right]$$

$$\frac{1}{\pi^2}\int_0^{\pi}\left[\cos(y)-\cos(\pi+y)\right]dy$$

$$\frac{1}{\pi^2}\left[\sin y-\sin(\pi+y)\right]_0^{\pi}$$

$$\frac{1}{\pi^2}\left[0-0\right]=?$$

cancelled +vol - same vol

38) Use Fubini's Thm to evaluate

$$\int_0^1\int_0^3 xe^{xy}\,dx\,dy = \int_0^3\int_0^1 xe^{xy}\,dy\,dx$$

$$\downarrow$$

$$\int_0^3 xe^{xy}\,dx$$

$$\left(x\frac{e^{xy}}{y} - \int \frac{e^{xy}}{y}\,dx\right)$$

inside $\int$ → $$\left[\frac{xe^{xy}}{y} - \frac{e^{xy}}{y^2}\right]_0^3 = \left(\frac{3e^{3y}}{y} - \frac{e^{3y}}{y^2}\right) - \left(-\frac{1}{y^2}\right)$$

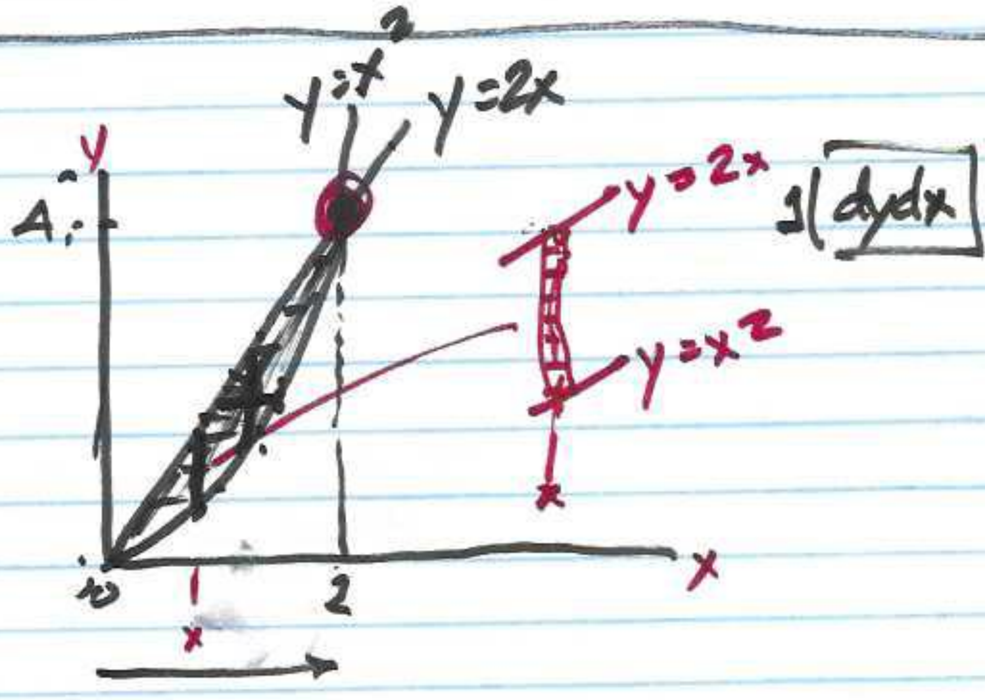
outside $\int$ $$\int_0^1 \left(\frac{3e^{3y}}{y} + \frac{1}{y^2} - \frac{e^{3y}}{y^2}\right)dy$$

5

$$\text{Vol} = \int_0^1 \int_0^x (3-x-y)\,dy\,dx$$

$$= \int_0^1 \left[3y - xy - \frac{y^2}{2}\right]_0^x dx = \int_0^1 \left(3x - x^2 - \frac{x^2}{2}\right) dx$$

$$\left[3x - \frac{3}{2}x^2\right]_0^1 = 3 - \frac{3}{2} = \boxed{\frac{3}{2}}$$



6 $\quad 1\,dy\,dx \quad \square^{dy}_{dx}$

$$\text{Area} = \int_0^2 \int_{x^2}^{2x} 1 \; dy\,dx$$

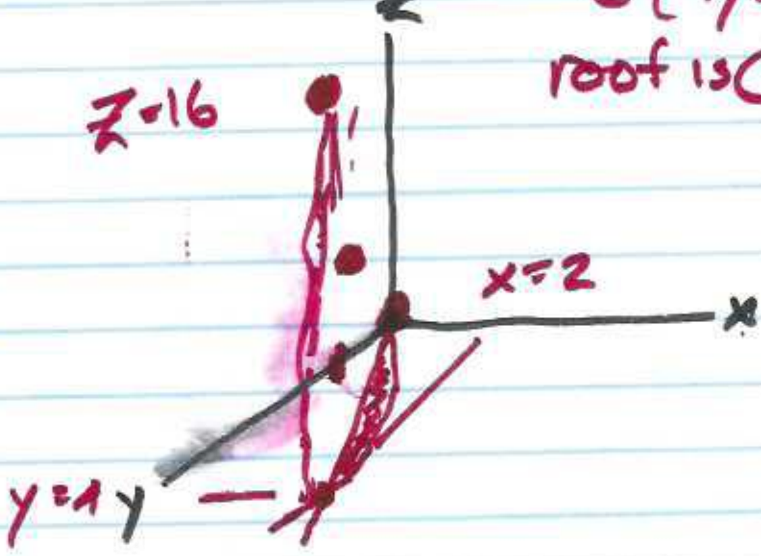
$$= \int_0^2 \Big[y\Big]_{x^2}^{2x} dx = \int_0^2 \Big[2x - x^2\Big] dx$$

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - 8/3 = \boxed{1\tfrac{1}{3}}$$

Put roof over meniscus:

Let $z = 2x + 3y$

@ $(x, y) = (2, 4)$
roof is @ 16



⑦

"z"

$$\int_0^2 \int_{x^2}^{2x} (2x+3y)\,dy\,dx = \underline{\underline{VOL}}$$

$$\int_0^2 \left[2xy + \frac{3y^2}{2}\right]_{x^2}^{2x} dx =$$

$$\int_0^2 \left[\left(4x^2+6x^2\right) - \left(2x^3 + \frac{3x^4}{2}\right)\right] dx$$

$$= \int_0^2 \left(10x^2 - 2x^3 + \frac{3}{2}x^4\right) dx$$

$$\left[\frac{10}{3}x^3 - \frac{\cancel{2}x^4}{\cancel{4}2} + \frac{3x^5}{10}\right]_0^2$$

$$\frac{10 \cdot 8}{3} - 8 + \frac{96}{10} = \underline{\qquad}$$

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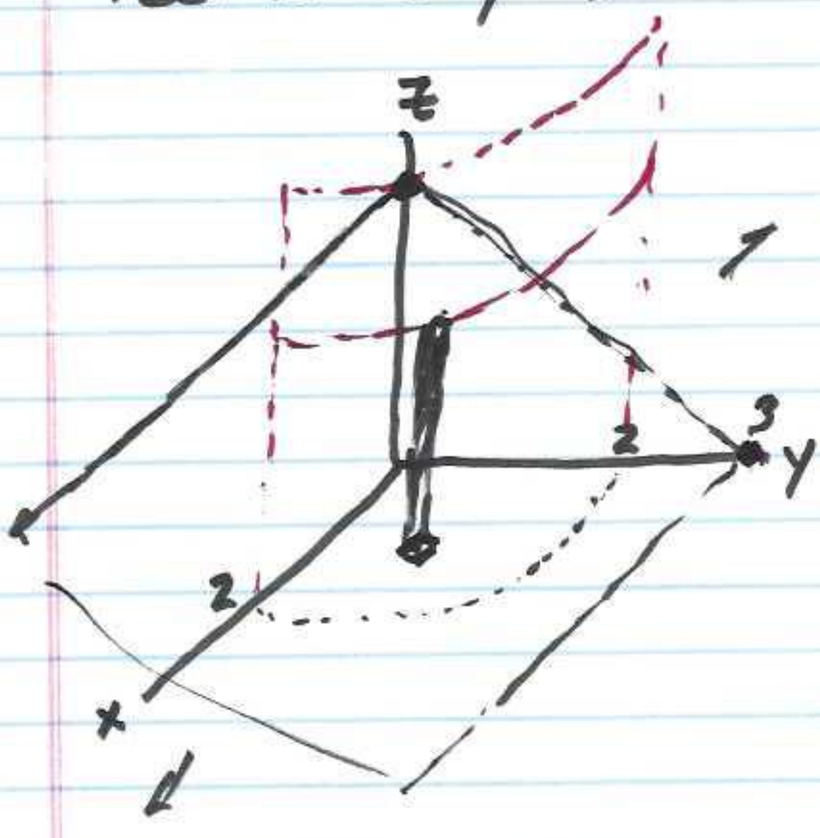
Walls are co-ordinate planes

Front wall $x^2+y^2=4$

Roof is $z+y=3$



$\int e^{x^2}dx$ NO

$\int \frac{dx}{\ln x}$ NO

Give Up — Use Fubini

Use $\displaystyle\int_0^3 \int_0^1 xe^{xy}\,dy\,dx$

$$\int_0^1 xe^{xy}\,dy = \left[\frac{\cancel{x}e^{xy}}{\cancel{x}}\right]_0^1 = \underline{e^x - 1}$$

2nd $\int$

$$\int_0^3 (e^x - 1)\,dx = \left[e^x - x\right]_0^3 = (e^3 - 3) - (1)$$

$$\boxed{e^3 - 4}$$ (this order much easier!)

Integrals over non-rectangular regions