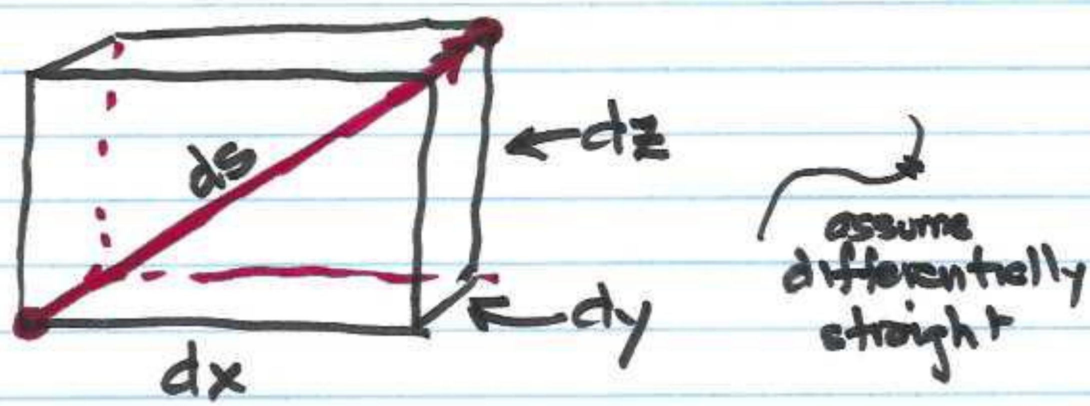
C4KY3

① Practice Quiz posted
② Scope of Exam Fri. Ch 12 thru 13.5 skip 6

<u>13.3</u>

Length of Space Curves



$$ds^2 = dx^2 + dy^2 + dz^2$$

so... $ds = \sqrt{dx^2 + dy^2 + dz^2}$

$r(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$

as diff. $\quad x'(t)dt \quad y'(t)dt \quad z'(t)dt$

$\quad dx \quad dy \quad dz$

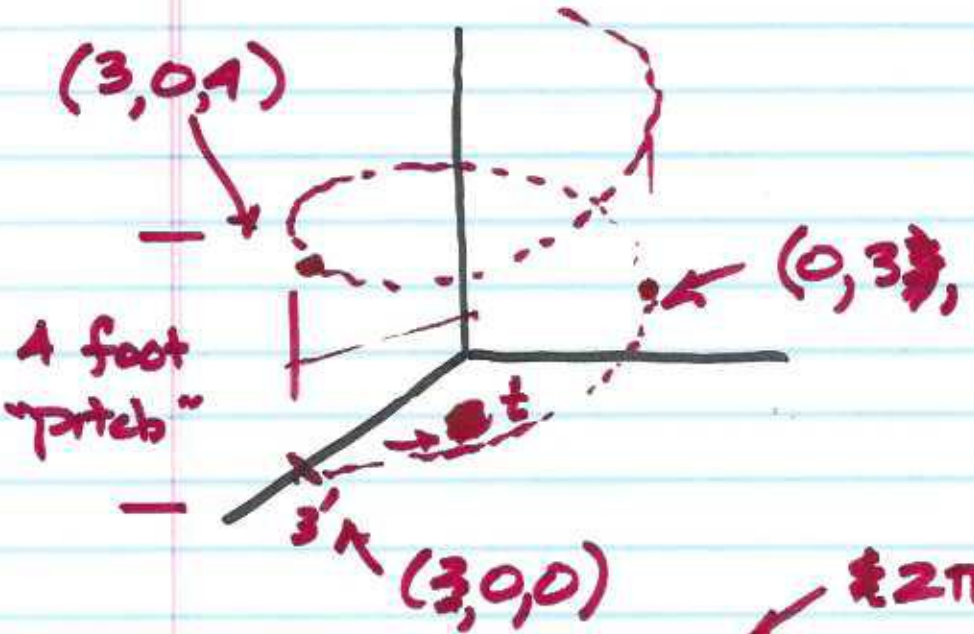
Now.. $ds = \sqrt{(x'(t)dt)^2 + (y'(t)dt)^2 + (z'(t)dt)^2}$

Also: $ds = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt$

Find length of curve from t_0 to t_f

$$L = \int_{t_0}^{t_f} ds$$

Spiral Stair 6' diam, rises 20' in 5 cycles



$\frac{t}{2\pi} \leftarrow 2\pi t$

$r(t) = 3\cos\frac{t}{2\pi} + 3\sin 2\pi t + 4t$

$r(t) = 3\cos 2\pi t\,\hat{i} + 3\sin 2\pi t\,\hat{j} + 4t\,\hat{k}$

(3)

$$\int_0^5 |r'(t)|\,dt = \int_0^5 \sqrt{\left(3(-\sin 2\pi t)2\pi\right)^2 + \left(3\cdot 2\pi\cdot\cos 2\pi t\right)^2 + (4)^2}\cdot dt$$

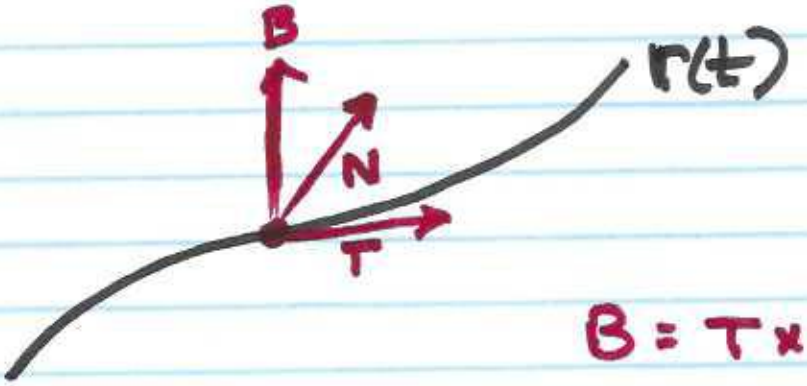
$$ds = \sqrt{36\pi^2\sin^2 2\pi t + 36\pi^2\cos^2 2\pi t + 16}\,dt$$

$$ds = \sqrt{36\pi^2 + 16}\,dt$$

So finally $L = \int_0^5 \sqrt{36\pi^2+16}\,dt$

$$2\int_0^5 \sqrt{9\pi^2+4}\,dt$$

$$= 2\int_0^5 9.63\,dt \quad \sim \underline{96.3}$$



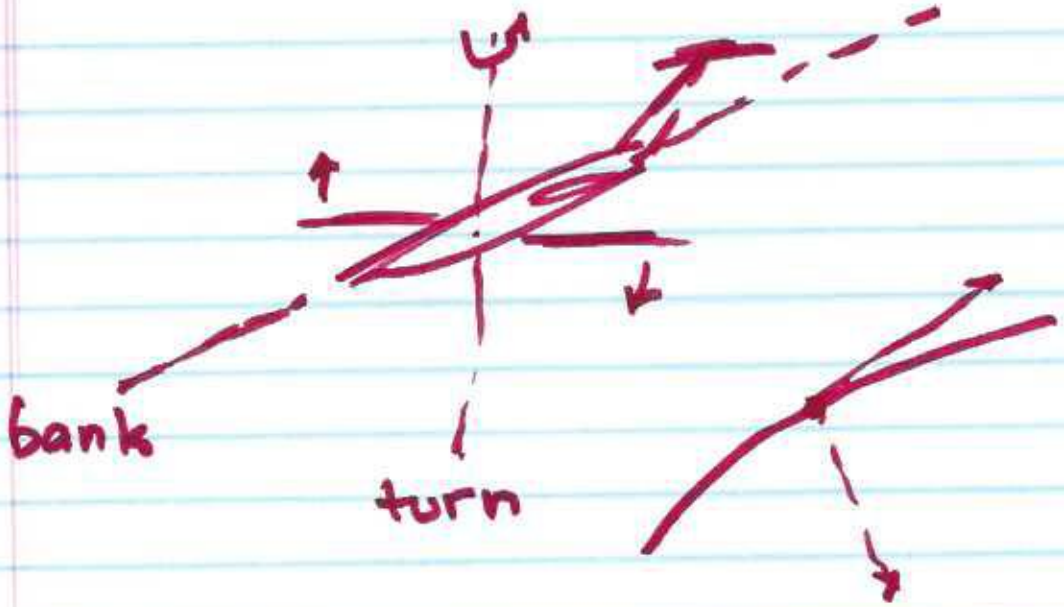
$B = T \times N$

T = unit tangent vector

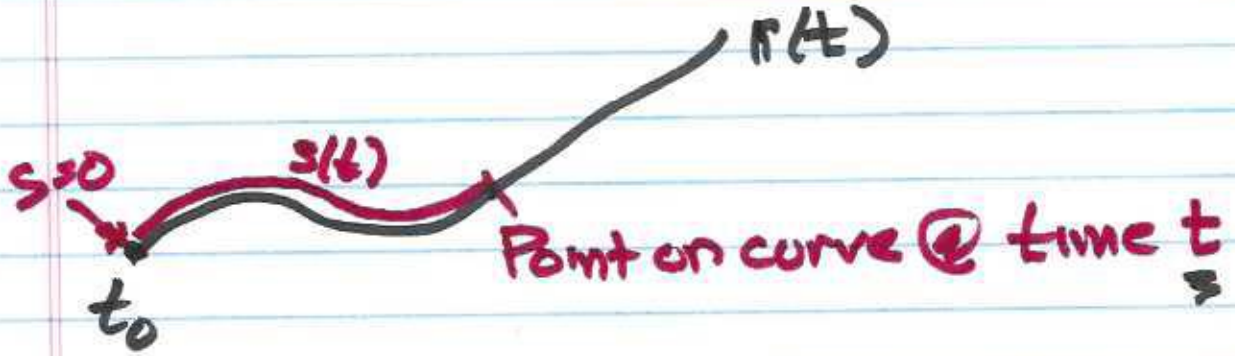
N = unit normal vector

B = unit binormal vector

TNB system is mini-xyz system that travels along curve



$$T := \frac{r'(t)}{|r'(t)|} \quad \text{(unit tangent)}$$



τ "tau" for t

$$s(t) = \int_{t_0}^{t} \sqrt{[x'(\tau)]^2 + [y'(\tau)]^2 + [z'(\tau)]^2}\, d\tau$$

$s'(t)$ = speed so $\frac{ds}{dt}$ is scalar

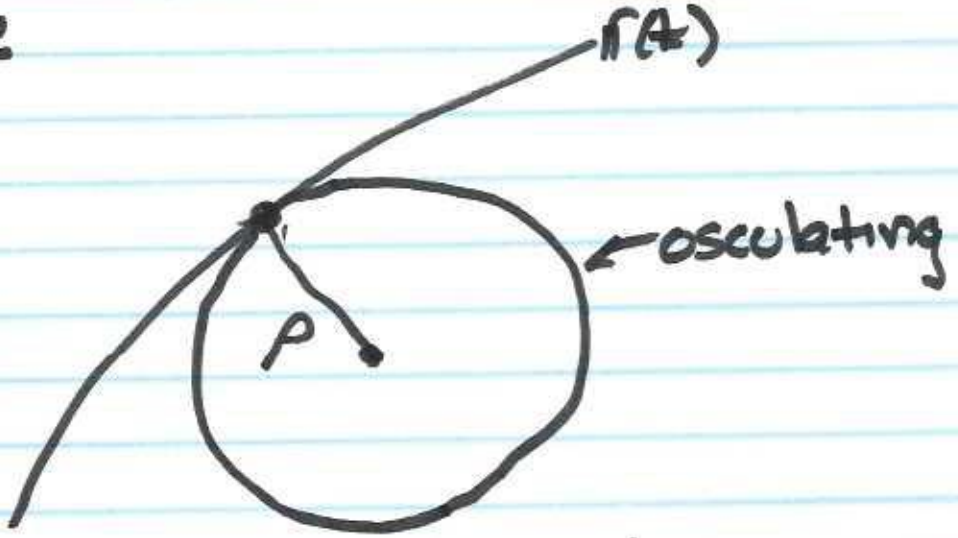
$$\frac{d\mathbf{r}}{ds} = \frac{d\mathbf{r}}{dt}\cdot\frac{dt}{ds}$$

$\frac{d\mathbf{r}}{dt}$ ↑ vector velocity

$\frac{dt}{ds}$ ↖ inverse of $s'(t)$

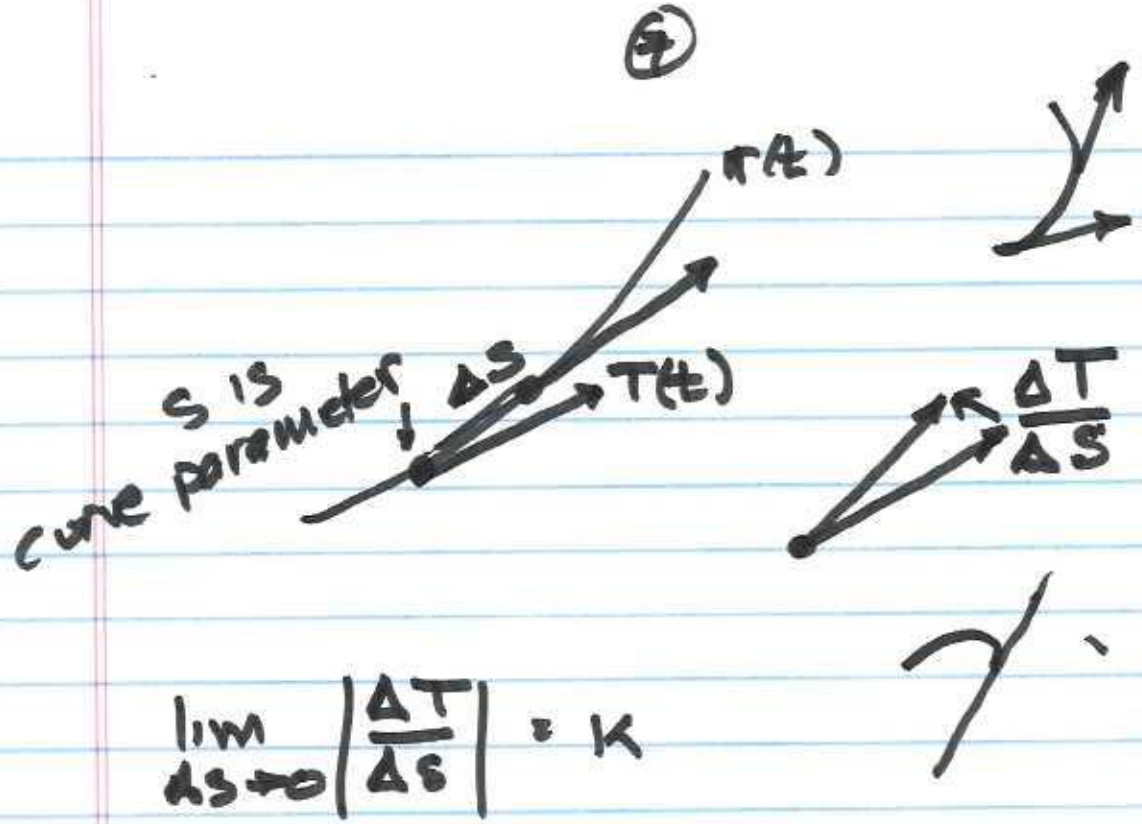
$\mathbf{v}\cdot\frac{1}{|\mathbf{v}|}$ = unit tangent $\mathbf{T}$.

Curvature



radius of curvature of $\mathbf{r}(t)$

curvature itself is defined as κ (kappa)



$$\lim_{\Delta s \to 0} \left| \frac{\Delta T}{\Delta s} \right| = \kappa$$

Calculating Formula (in terms of t)

$$\kappa = \frac{1}{|v|} \cdot \left| \frac{dT}{dt} \right|$$

Def^{n} $N = \frac{1}{\kappa} \frac{dT}{ds}$

Calculating $N = \dfrac{dT/dt}{|dT/dt|}$ ←

(3)

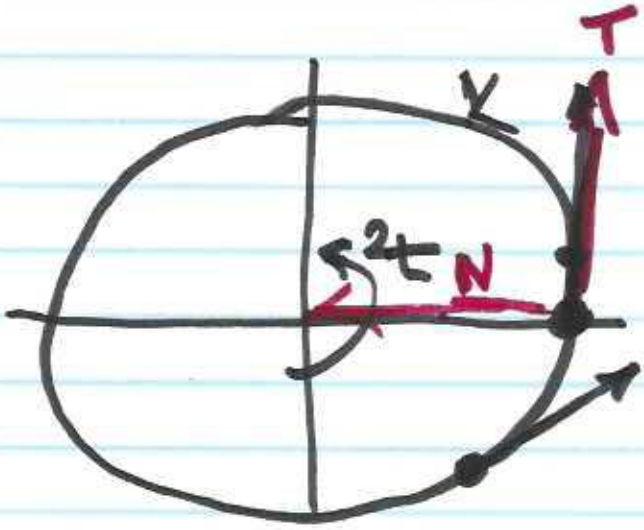
Given $r(t) = \cos 2t\,\hat{i} + \sin 2t\,\hat{j}$

Find T:

$$v(t) = r'(t) = -2\sin 2t\,\hat{i} + 2\cos 2t\,\hat{j}$$

$$|v(t)| = \sqrt{4\sin^2 2t + 4\cos^2 2t} = 2$$

$$T = -\sin 2t\,\hat{i} + \cos 2t\,\hat{j}$$



Find N: $\frac{dT}{dt} = \frac{-2\cos 2t\,\hat{i} + 2\sin 2t\,\hat{j}}{2}$

$$= -\cos 2t\,\hat{i} + \sin 2t\,\hat{j}$$