Q9QSS

①

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Review: A, B vectors

① $A \cdot B = |A||B|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3 = \sum_{i=1}^{3} a_ib_i$

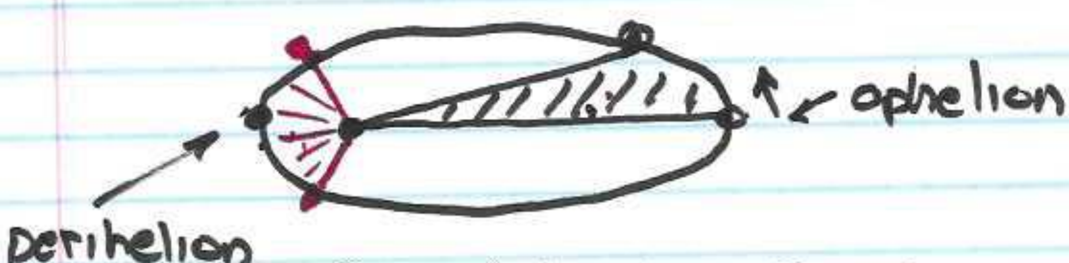
$A \cdot B = 0$ implies $A \perp B$

② $A \times B = |A||B|\sin\theta \, e_\perp$ ← unit vector ⊥ to A-B plane

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$A \times B = 0$ implies $A \parallel B$ or antiparallel

Let's prove Kepler's Second Law



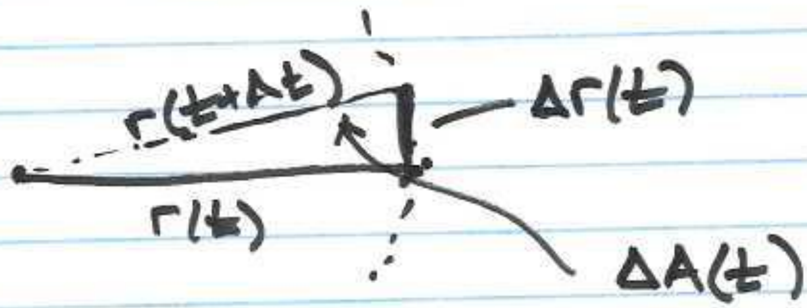
Rate of Area being Swept = constant

sides

Convenient area formula $\text{Area} = \frac{1}{2}|A \times B|$

To show A = area being swept (by radius)

$\rightarrow$ $A'(t)$ = rate of sweeping (constant per Kepler) so show $A''(t) = 0$.



$$\Delta A(t) = \frac{1}{2}\left| r(t) \times \Delta r(t) \right|$$

$$A'(t) = \frac{\Delta A(t)}{\Delta t} = \frac{1}{2}\left| r(t) \times \frac{\Delta r(t)}{\Delta t} \right|$$

As $\Delta t \to 0$ we have $A'(t) = \frac{1}{2}\left| r(t) \times r'(t) \right|$

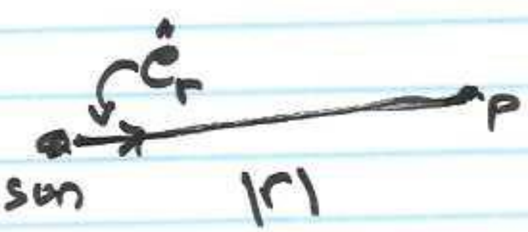
Now calculate $A''(t)$

So $A''(t) = \frac{1}{2}\left[\underbrace{|r'(t) \times r'(t)|}_{\substack{0 \\ \text{(same vector)}}} + \underbrace{|r(t) \times r''(t)|}_{\substack{0 \\ \text{(see below)}}}\right]$

What about $r(t) \times r''(t)$

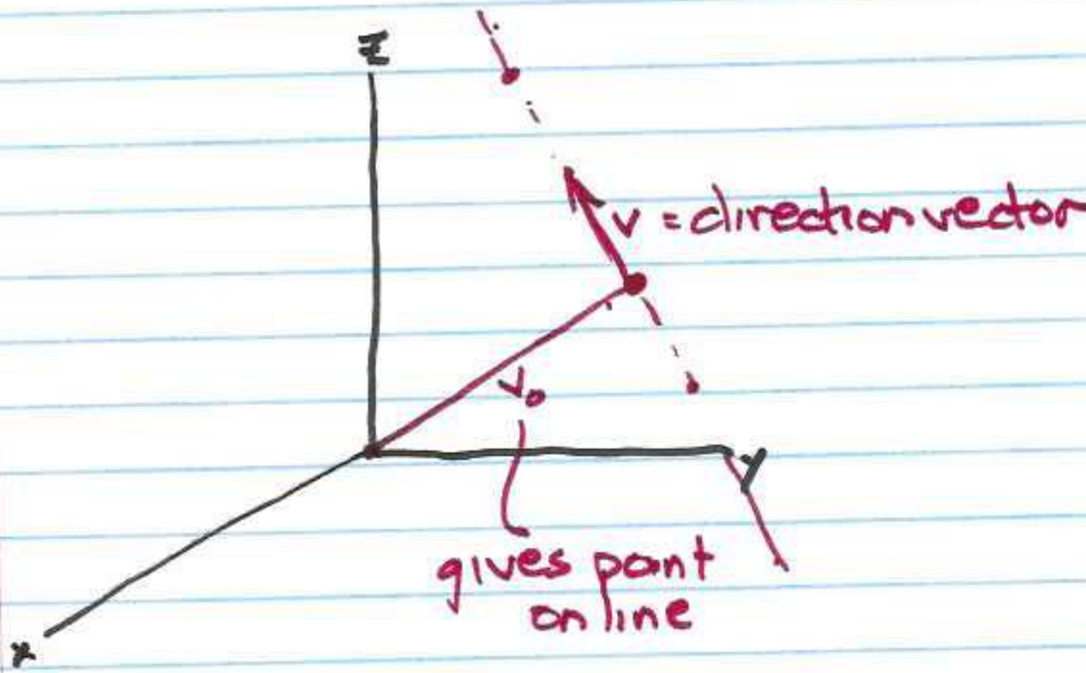
$\rightarrow F = m r''(t)$

(Newton's) Formula $F = -\frac{G M_s M_p}{|r|^2}\hat{e}_r = M_p r''(t)$



Finally $A''(t) = 0 \rightarrow \underbrace{A'(t) = 0}_{\text{Kepler 2}}$

(1)



Eqn of line in space is $r(t) = v_0 + t v$

scalar (think time)

Standard parametrization of line:

$v_0 = \langle x_0, y_0, z_0 \rangle$, $v = \langle x, y, z \rangle$

$x(t) = x_0 + tx$, $y(t) = y_0 + ty$, $z(t) = z_0 + tz$

Prob: Find parametric equations for line that goes thru $P(-3,2,-3)$ and $Q(1,-1,4)$

Set $\mathbf{v}_0 = \langle -3, 2, -3 \rangle$, now find $\mathbf{v}$

$\overrightarrow{QP} = \langle 4, -3, 7 \rangle = \mathbf{v}$

Vectorially $\mathbf{r}(t) = \mathbf{v}_0 + t\,\mathbf{v} = ?$

$\mathbf{r}(t) = \langle -3, 2, -3 \rangle + t \langle 4, -3, 7 \rangle$

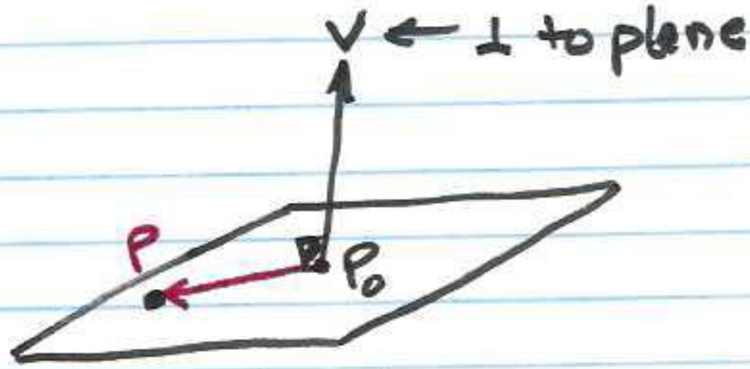
Now find std parametrization:

$x(t) = -3 + t(4)$

$y(t) = 2 + t(-3)$

$z(t) = -3 + 7t$

Now: planes



Note $(P-P_0)\cdot V = 0$

Claim: this is an equation for plane

(i.e. all points lying in plane)

If you prefer, we can make V a "unit"

vector $\hat{n}$ = unit outward normal

Ex: Find an eqn for plane through $P_0(-3,0,7)$.

perpendicular to $\hat{n} = \dfrac{\langle 5,2,-1\rangle}{\sqrt{30}}$.

Let $(x,y,z) = P$ (arbitrary point in plane)

So $\langle x+3, y, z-7\rangle \cdot \langle 5,2,-1\rangle = 0$

$5x + 15 + 2y - (z-7) = 0$

$5x + 2y - z = -22 \leftarrow$ plane

Suppose $ax+by+cz=d$ is plane eqn.

What is normal (direction) vector.

$\langle a, b, c \rangle$ - done.

Ex: $7x-3y+z=16$ (plane #1)

$14x-6y+2z=0$ (" #2)

are they parallel?

Quadric Surfaces - memorize forms

Given eqn: $Ax^2+By^2+Cz^2+Dz=E$

this is always a so-called "quadric" surface

Curves in Space $\mathbf{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$

$\langle a,b,c \rangle = a\hat{i}+b\hat{j}+c\hat{k}$

"component functions"

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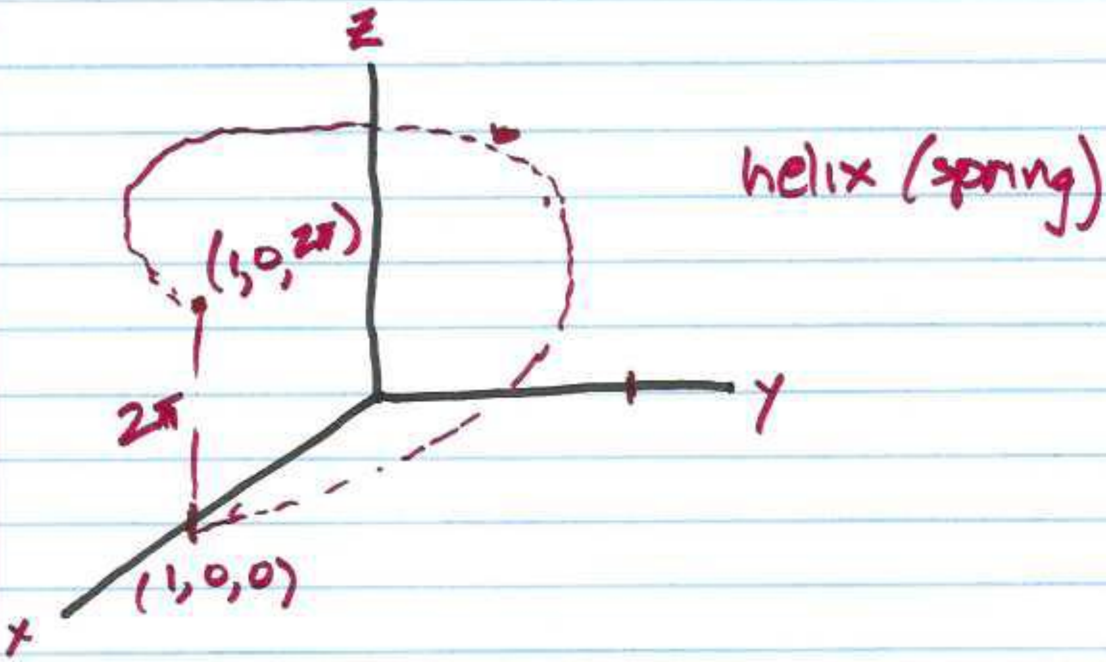

$\mathbf{r}(t) = (\cos t)\,\hat{i} + (\sin t)\,\hat{j} + t\,\hat{k}$

$t \in [0, 2\pi]$