QV5Y7

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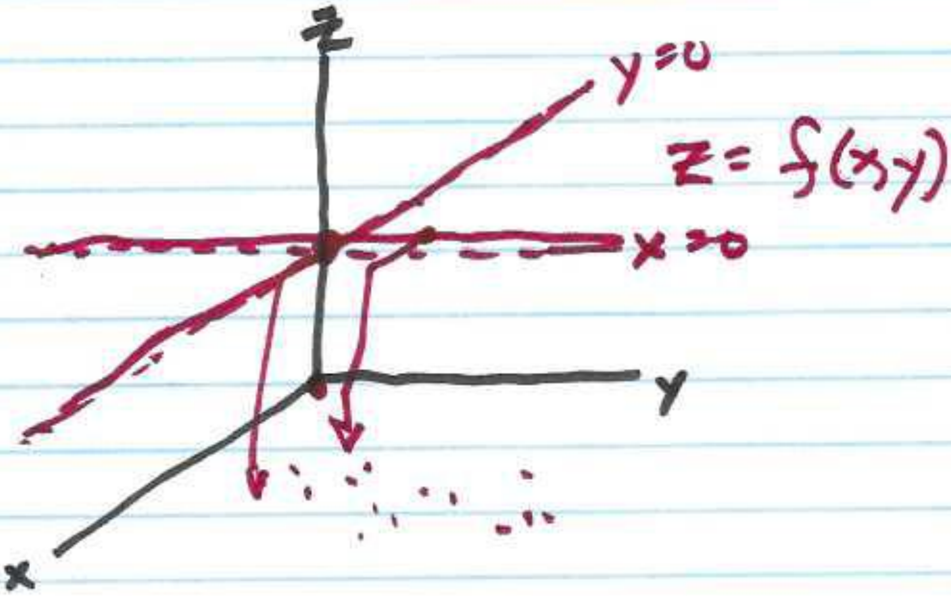
Partial Derivatives

Does existence of partial derivatives guarantee function is continuous.



$$f(x,y) = 0 \text{ if } x,y \neq 0$$
$$= 1 \text{ if } x=0 \text{ or } y=0$$

Notation:

3 ways to denote partials:

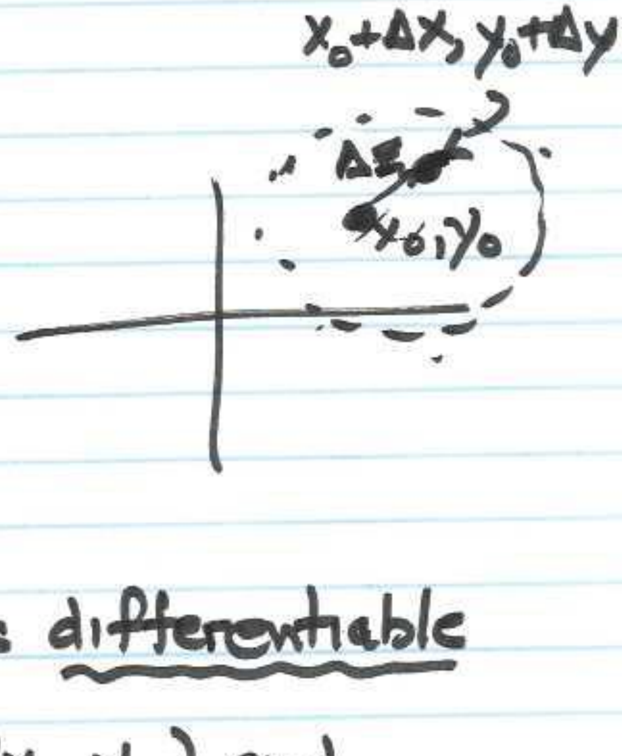
$$f(x,y) \Rightarrow f_x,\ \frac{\partial f}{\partial x} = \frac{\partial f(x,y)}{\partial x},\ D_x f$$

$$f_y = -2xyz + x^2$$

$$(f_y)_x = -2yz + 2x$$

$$(f_{yx})_y = -2z$$

$$(f_{yxy})_z = -2$$



Def^n

$z =$

Function $f(x,y)$ is differentiable

@ (x_0,y_0) if $f_x(x_0,y_0)$ and

$f_y(x_0,y_0)$ exist and

$$\Delta f = \Delta z = f(x_0+\Delta x, y_0+\Delta y) - f(x_0,y_0)$$

must satisfy this eqn:

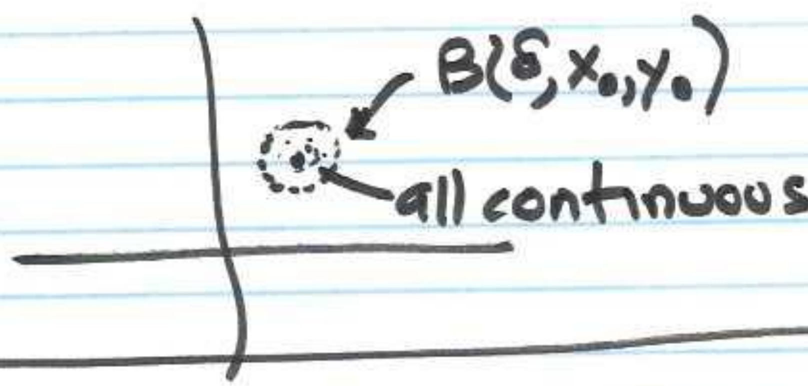
$$\Delta z = f_x(x_0,y_0)\Delta x + f_y(x_0,y_0)\Delta y + \underline{\varepsilon_1 \Delta x + \varepsilon_2 \Delta y}$$

Clairaut's Th^{m} (Mixed derivatives)

(i) Applies to $f(x,y)$

(ii) If $f(x,y)$, f_x, f_y, f_{xy} & f_{yx} are continuous in some neighborhood of $f(x_0,y_0)$, then $f_{xy} = f_{yx}$ @ (x_0,y_0).

Extension to 3 or more variables.



Find f_{yxyz} if $f(x,y,z) =$?

$$1 - 2xy^2z + x^2y$$

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$$\text{1}^{\text{st}}\text{ partials}\left\{\begin{array}{l} w_x = \dfrac{1}{2x+3y}\cdot(2) = \dfrac{2}{2x+3y} \\[2ex] w_y = \dfrac{1}{2x+3y}(3) = \dfrac{3}{2x+3y} \end{array}\right.$$

$$w_{xy} = \left[2(2x+3y)^{-1}\right]_y =$$

$$2\left(-(2x+3y)^{-2}\cdot 3\right) = \frac{-6}{(2x+3y)^2}$$

$$w_{yx} \quad \left[3(2x+3y)^{-1}\right]_x =$$

same

$$3\left(-(2x+3y)^{-2}\cdot 2\right) = \frac{-6}{(2x+3y)^2}$$

$$\frac{\partial R}{\partial R_1} = \underline{\frac{R_2}{R_1+R_2} + \frac{R_1R_2}{(R_1+R_2)^2}}$$

$$R = (R_1R_2)\cdot(R_1+R_2)^{-1}$$

$$\frac{\partial R}{\partial R_1} = (R_2)(R_1+R_2)^{-1} + (R_1R_2)\cdot(R_1+R_2)^{-2}(1)$$

Multiple differentiations

55) $W = \ln(2x+3y)$

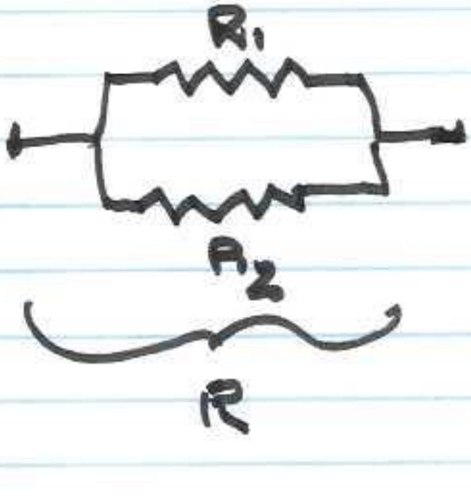
We want $W_x, W_y, \underbrace{W_{xy}, W_{yx}}_{?\ \text{same}}$

34) $f(x,y,z) = \sinh(xy - z^2)$

$f_x = \cosh(xy - z^2) \cdot (y)$

$f_y = \cosh(xy - z^2) \cdot (x)$

$f_z = \cosh(xy - z^2) \cdot (-2z)$



$R = R(R_1, R_2)$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R} = \frac{R_1 + R_2}{R_1 R_2} = \frac{1}{R}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$f(x,y) = \frac{xy}{x+y}$

14) $f(x,y) = e^{-x}\sin(x+y)$

$$f_x = -e^{-x}\sin(x+y) + e^{-x}\cos(x+y)(1)$$

$$= e^{-x}\left[\cos(x+y) - \sin(x+y)\right]$$

$$f_y = e^{-x}\cos(x+y)(1)$$

27) $f(x,y,z) = \arcsin(xyz)$

$$f_x = \frac{yz}{\sqrt{1-(xyz)^2}}$$

$$f_y = \frac{xz}{\sqrt{1-(xyz)^2}}$$

$$\arcsin(kx)' = \frac{k}{\sqrt{1-k^2x^2}}$$

$$f_z = \frac{xy}{\sqrt{1-(xyz)^2}}$$

if you are sure that all partial derivateses

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

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1) $f(x,y) = 2x^2 - \underline{3y} - 4$

$$\frac{\partial f}{\partial x} = 4x - 0 - 0 = 4x$$

$$\frac{\partial f}{\partial y} = -3$$

7) $f(x,y) = \sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$

$$\frac{\partial f}{\partial x} = \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$