

TC3MN

①

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T/F Practice:

F 1) $f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right)$

$\uparrow$ $\leftarrow$ $\times$ $\downarrow$
 $\rightarrow$ 2nd 1st

T 2) $f_{xy} = f_{yx}$ always sometimes

F 3) sign of f_{xx} & f_{yy} are always the same

$\rightarrow \Delta = \underline{f_{xx} f_{yy}} - f_{xy}^2 > 0$

F 4) If f_x & f_y exist at a point, is f differentiable?

T 5) Is $\underline{D_{xyz} f} = f_{xyz}$

T 6) Is tangent plane $\perp$ to the normal line @ a point on surface $z = f(x, y)$

(2)

7) Is the linearization of $f(x, y)$ @ (x_0, y_0)

$$\rightarrow \Delta f(x, y) \approx \left. \frac{\partial f}{\partial x} \right|_{x_0} \Delta x + \left. \frac{\partial f}{\partial y} \right|_{y_0} \Delta y$$

Analogue in 1-D $f(x) - f(x_0) \approx f'(x_0) \Delta x$

$\underbrace{f(x) - f(x_0)}_{\Delta f(x_0)}$

8) $\underbrace{\nabla \phi(x, y, z)}_{\text{scalar function}} = \left\langle \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right\rangle$

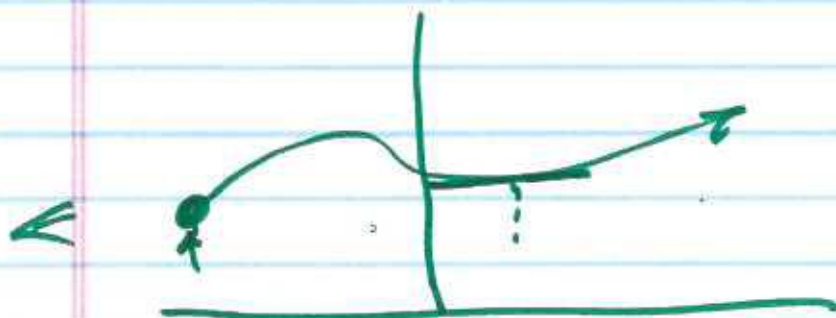
9) $D_u f(x, y) = \left. \nabla f \right|_{(x_0, y_0)} \cdot u = |\nabla f| \cos \Theta$

10) Gradient - max positive change

11) Interior pt of set



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$$\underline{z = f(x, y)}$$

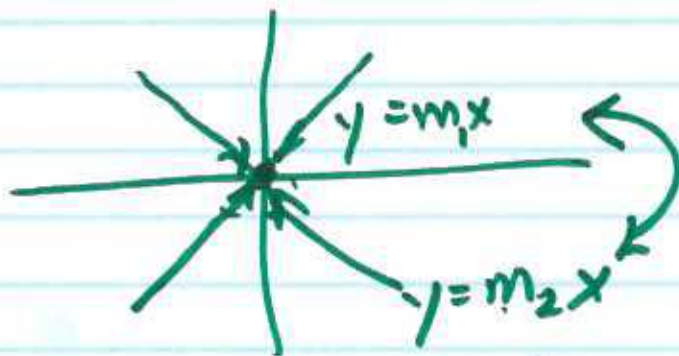


$$z = \text{const.}$$

$$12) \nabla(fg) = ? \quad f \nabla g + g \nabla f$$

$$13) \nabla\left(\frac{f}{g}\right) = \frac{g \nabla f - f \nabla g}{g^2}$$

$$14) \frac{x^2 y}{x^4 + y^2}$$

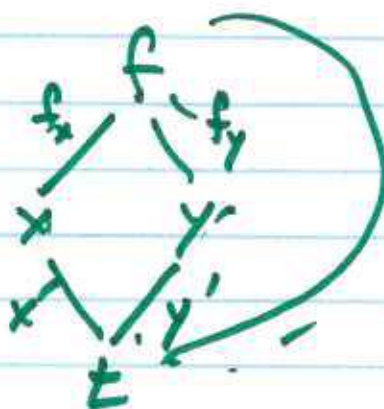


$$15) f(x, y), x(t), y(t) \text{ find deriv w/r } t$$

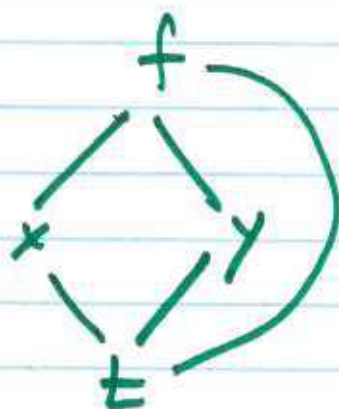
$$d \circ$$

$$\rightarrow \frac{\partial f}{\partial t}$$

$$\frac{df}{dt}$$



$$\underline{f(x, y, t)} \quad x = x(t) \quad y = y(t)$$

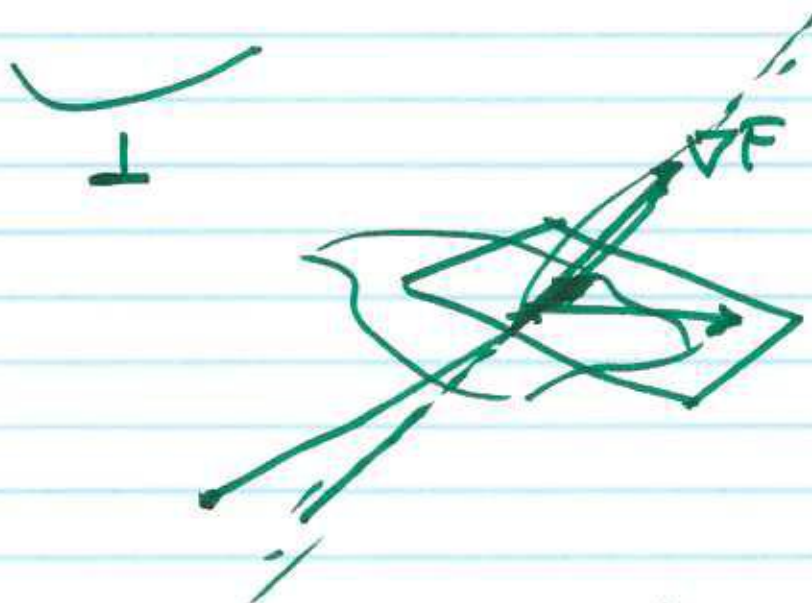


Given $\boxed{f(x, y) = z}$ $\rightarrow \Rightarrow F(x, y, f(x, y)) = 0$ @ point P_0 on surface
 what is eqn of tangent plane?

$$\nabla f = \langle f_x, f_y \rangle$$

$$16) \quad \nabla F|_{P_0} = \langle f_x, f_y, -1 \rangle$$

$$\nabla F|_{P_0} \cdot \langle x-x_0, y-y_0, z-z_0 \rangle = 0$$



$$\langle x_0, y_0, z_0 \rangle + \pm \nabla F|_{P_0}$$

17) What is the Hessian determinant?

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \Delta$$

18) ~~$f(x,y,z)$~~ $g(x,y,z)$

$f(x,y,z)$ is obj function

$g(x,y,z)$ is constraint

Form auxiliary function

Take (Lagrange multiplier) $\lambda = \text{constant}$

$$\underline{f + \lambda g} \text{ or } \lambda f + g = \phi(x,y,\lambda)$$

Find $\phi_x = 0$

$$\phi_y = 0$$

$$\phi_\lambda = g(x,y,z) = 0$$

} solve
simultaneously