E4RCV

(1)

Review line integral

$$\int_C f(x,y,z)\,ds \longrightarrow \int_C f(x(t),y(t),z(t))\left|\frac{d\mathbf{r}(t)}{dt}\right| dt$$

↑ express parametrically

calc formula

Note $ds = \left|\frac{d\mathbf{r}(t)}{dt}\right| dt$

↑ velocity

Vector Fields:

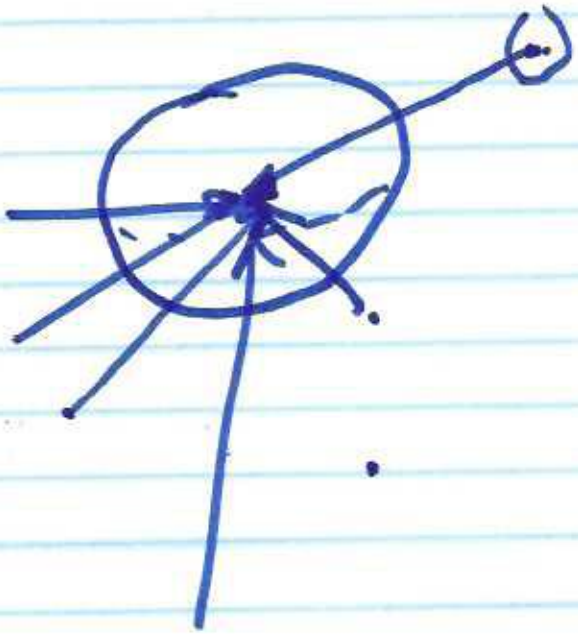
$\phi: \mathbb{R}^3 \to \mathbb{R}^3$

$f(x,y,z) \longrightarrow \nabla f(x,y,z) =$

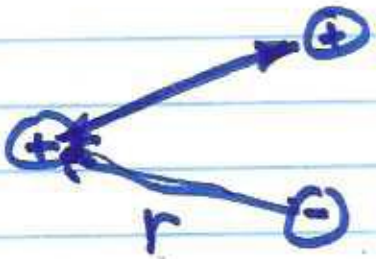
$$\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

typical vector in a vector field

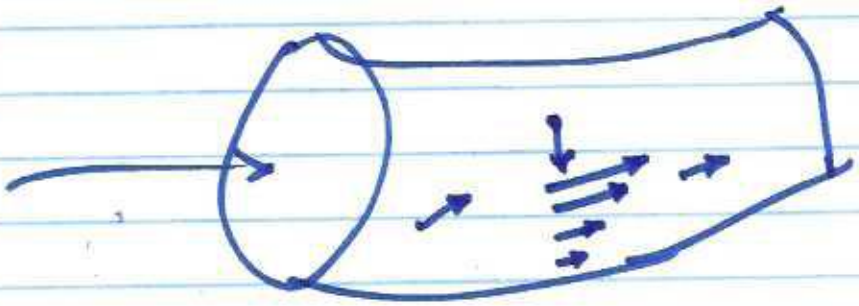
$$\vec{F} = -\frac{GMm}{r^3}\,\vec{r} \qquad \vec{r} = r\hat{e}$$

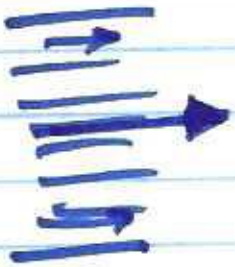


(3)



E(x, y, z)





$$\int_C \mathbf{F}\cdot \underbrace{\mathbf{r}(t)\,dt}_{d\mathbf{r}} = \int_C \mathbf{F}\cdot d\mathbf{r}$$

Ex. $\mathbf{F}(x,y,z) = z\hat{i} + xy\hat{j} - y^2\hat{k}$

Let curve C be $\mathbf{r}(t) = \underbrace{t^2}_{x}\hat{i} + \underbrace{t}_{y}\hat{j} + \underbrace{\sqrt{t}}_{z}\hat{k}$

$t \in [0,1]$

What is $\int_C \mathbf{F}\cdot d\mathbf{r}$ $\longrightarrow$ $\frac{d\mathbf{r}}{dt}\,dt$

$$\frac{d\mathbf{r}}{dt} = 2t\hat{i} + \hat{j} + \frac{1}{2\sqrt{t}}\hat{k}.$$

$$\mathbf{F}\cdot d\mathbf{r} = \left(t^{5/2} + t^4 - t^{5/2}\right) = t^4$$

$$\int_0^1 t^4\,dt = \frac{t^5}{5}\Big|_0^1 = \boxed{1/5}$$

$$F(t) = \sqrt{t}\,\hat{i} + t^2 \cdot t\,\hat{j} - t^2\hat{k}$$

OIS $\quad dr = \left(2t\,\hat{i} + \hat{j} + \frac{1}{2\sqrt{t}}\hat{k}\right)dt$

$$F(t)\cdot dr = 2t^{3/2} + t^2 - \frac{1}{2}t^{3/2}$$

$$\int_{t=0}^{1} \left(2t^{3/2} + t^2 - \frac{t^{3/2}}{2}\right)dt =$$

$$= \left[\frac{2}{5}\cdot 2t^{5/2} + \frac{t^3}{3} - \frac{2}{5}\cdot\frac{t^{5/2}}{2}\right]_0^1$$

$$\frac{4}{5} + \frac{1}{3} - \frac{1}{5} = \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$$