① 4/6

Line Integrals -

Reg. Riemann Integral

$$S_n = \sum_{i=1}^{\infty} \underline{f(x_i)} \Delta x_i$$

$$\frac{ds}{dt} = \left|\frac{dr}{dt}\right|$$

$$\underline{S = \int f(x)\,dx}$$

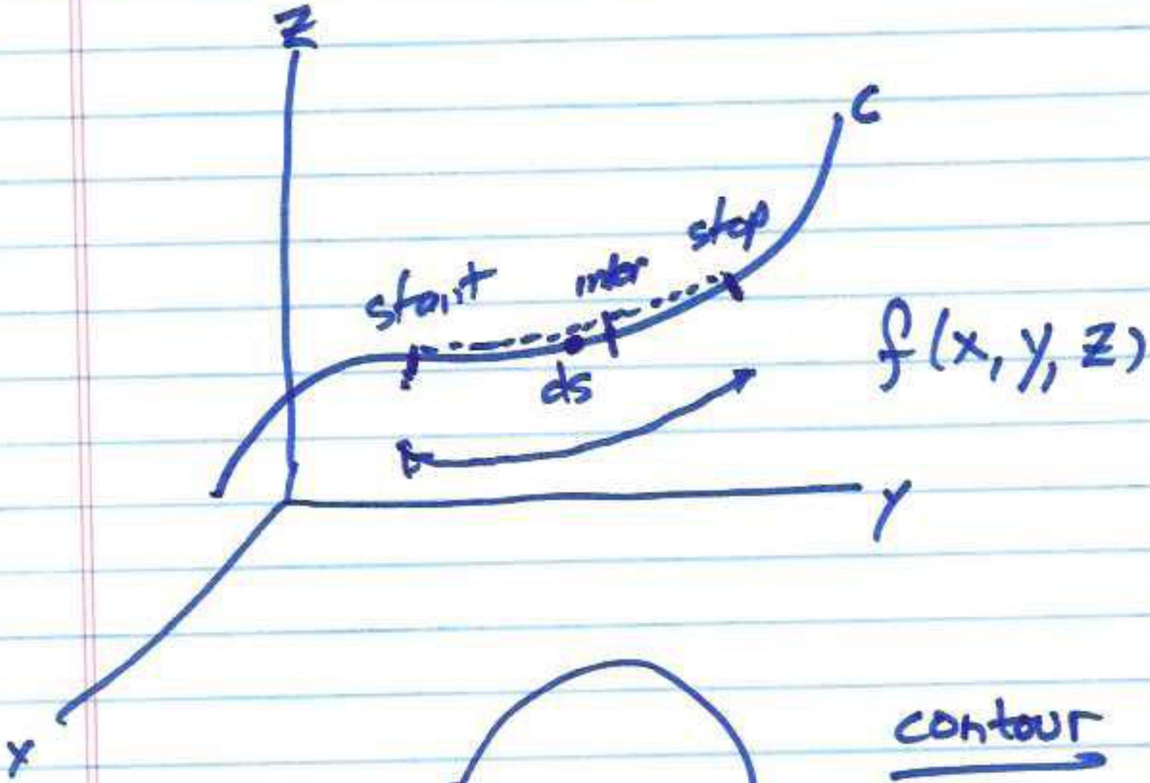
s means length along a line

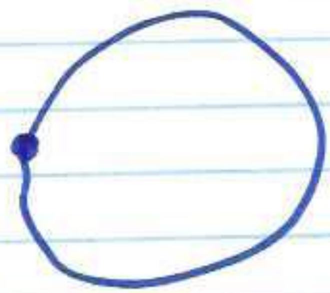
Line version

$$S_n = \sum_{k=1}^{n} f(x_k, y_k, z_k)\,\Delta s_k$$

$$\lim_{n\to\infty} \quad \underline{\int f(x,y,z)\,ds}$$

$$ds^2 = dx^2 + dy^2 + dz^2$$





contour

$\oint f\, ds$

To evaluate a line integral:

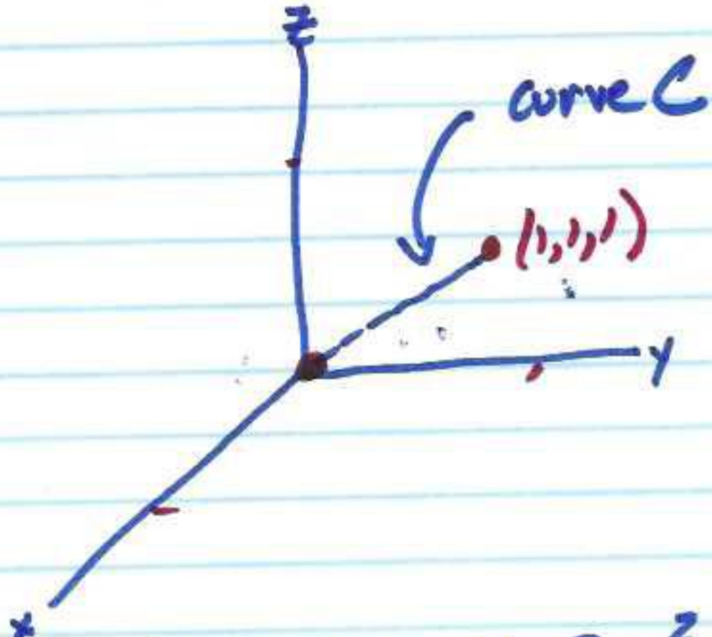
1) $\int f(x(t), y(t), z(t)) \left|\frac{ds}{dt}\right| dt$

parametrize

$v(t) = \left|\frac{dr}{dt}\right| = \frac{ds}{dt}$

2) Restate integral in terms of t & do.

③



$$f(x,y,z) = x - 3y^2 + z$$

$$\int_C f(x,y,z)\, ds$$

1) Parametrize: $\mathbf{r}(t) = t\hat{i} + t\hat{j} + t\hat{k}$

$t \in [0,1]$

$$ds = \left|\underset{\text{vel}}{\frac{d\mathbf{r}}{dt}}\right| dt \quad \text{so...} \quad ds = \sqrt{3}\, dt$$

$$\sqrt{3} \qquad \left|\frac{d\mathbf{r}}{dt}\right| = \left|\hat{i} + \hat{j} + \hat{k}\right| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

4

$$\int_0^1 \underbrace{(t-3t^2+t)}_{f(x,y,z)} \cdot \underbrace{\sqrt{3}\,dt}_{ds}$$

$$\left[\frac{t^2}{2} - t^3 + \frac{t^2}{2}\right]_0^1 = (t^2 - t^3)\Big|_0^1 = \underline{0}$$

Try another line: (same function)

Go from origin to $\underline{(1, \sqrt{2}, 0)}$.

$$\left.\begin{array}{l} \mathbf{r}(t) = t\,\hat{i} + \sqrt{2}t\,\hat{j} + 0\,\hat{k} \\ t \in [0,1] \end{array}\right\}$$

$$\int_0^1 (t - 3 \cdot 2t^2 + 0)\, ds$$

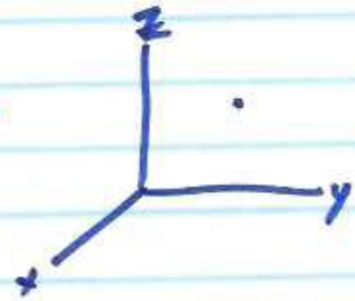
$$ds = \left|\frac{d\mathbf{r}}{dt}\right| dt$$

$$\mathbf{r}'(t) = 1\hat{i} + \sqrt{2}\hat{j} + 0\hat{k}$$

$$\left|\frac{d\mathbf{r}}{dt}\right| = \sqrt{3} \quad \text{so } ds = \sqrt{3}\,dt \text{ as before}$$

$$\int_0^1 (t - 6t^2)\sqrt{3}\,dt$$

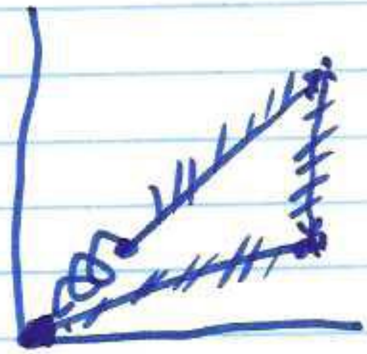
$$\sqrt{3}\left[\frac{t^2}{2} - 2t^3\right]_0^1 = \underline{-2\sqrt{3}}$$

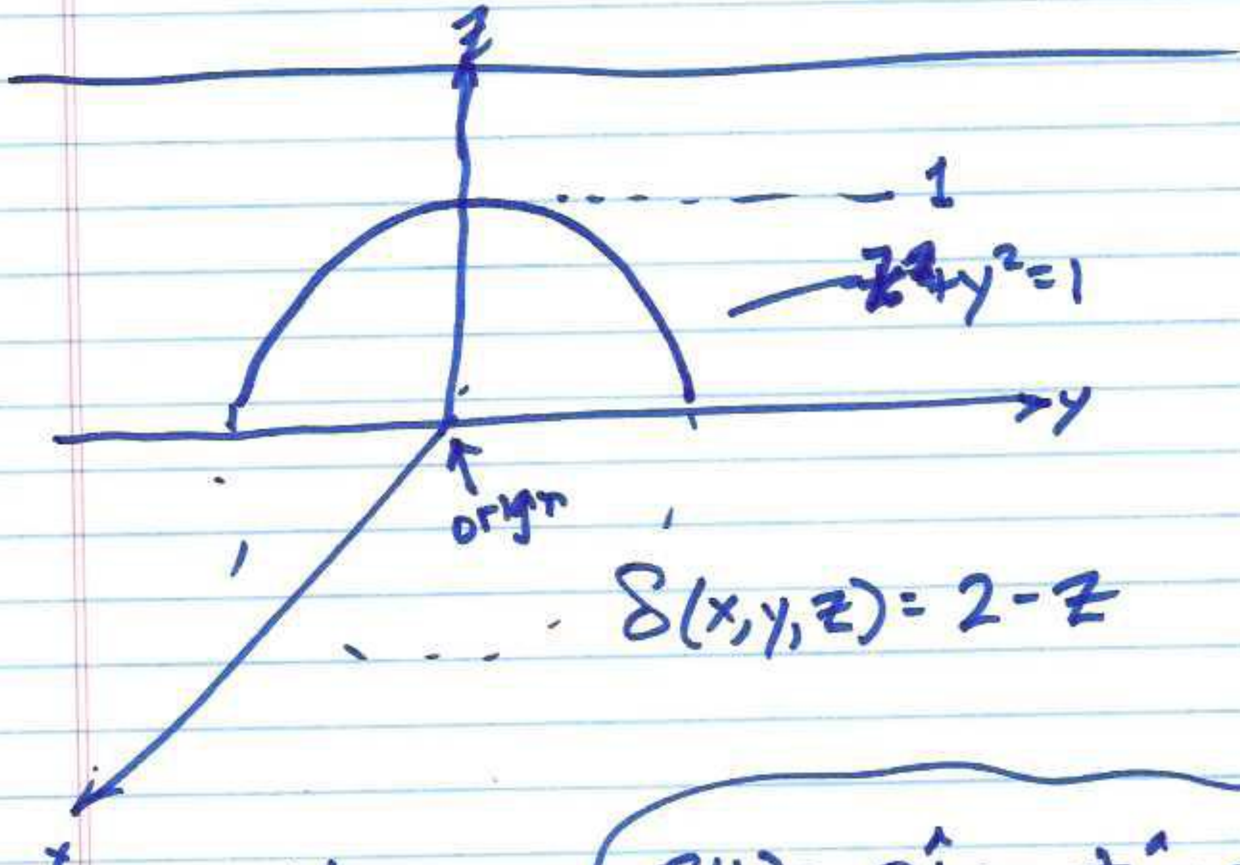


$$\sqrt{3}\left[\frac{t^2}{2} - 2t^3\right]_0^1 =$$

$$\sqrt{3}\left(\frac{1}{2} - 2\right) = -\frac{3\sqrt{3}}{2} \neq 0$$

so direction matters





$$\delta(x,y,z) = 2 - z$$

$$\bar{z} = \frac{M_{xy}}{M}$$

$$\vec{r}(t) = 0\hat{i} + \cos t\,\hat{j} + \sin t\,\hat{k}$$

$$t \in [0, \pi]$$

7

$$\mathbf{r}'(t) = 0\hat{i} - \sin t\,\hat{j} + \cos t\,\hat{k}$$

$$|\mathbf{r}'(t)| = \sqrt{\sin^2 t + \cos^2 t} = 1$$

$$ds = \underbrace{\left|\frac{d\mathbf{r}}{dt}\right|}_{1} dt = dt$$

$$M = \int_C \delta\, ds = \int_C (2-z)\, ds =$$

$$\int_0^{\pi} (2 - \overset{\sin}{\cancel{\cos}}\, t)\, dt = \Big[2t + \cos t\Big]_0^{\pi} =$$

$$\big(2\pi + (-1)\big) - \big(0 + 1\big) =$$

$$M = 2\pi - 2$$

$$\int_C (\sin t)(2-\sin t)\,ds$$

$\sin^2 t = \dfrac{1-\cos 2t}{2}$

$$M_{xy} = \int_0^{\pi} (2\sin t - \underline{\sin^2 t})\,dt =$$

$-2\cos t -$

Aside: $\frac{1}{2}\int(1-\cos 2t)\,dt = \frac{1}{2}\left[t - \frac{\sin 2t}{2}\right]_0^{\pi}$

$\frac{1}{2}\left[(\pi)-(0-0)\right]$ $\frac{\pi}{2}$

So... $M_{xy} = -2\cos t\Big|_0^{\pi} - \frac{\pi}{2}$

$(-2\cos t)\Big|_0^{\pi} = 2+2 = 4$

9

$$\frac{M_{xy}}{M} = \frac{4}{2\pi - 2}$$

Book: 0.57

Actual: $\frac{8 - \pi}{4\pi - 4}$