G8RD8

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Review:

$$\iiint f(x,y,z)\,dx\,dy\,dz$$

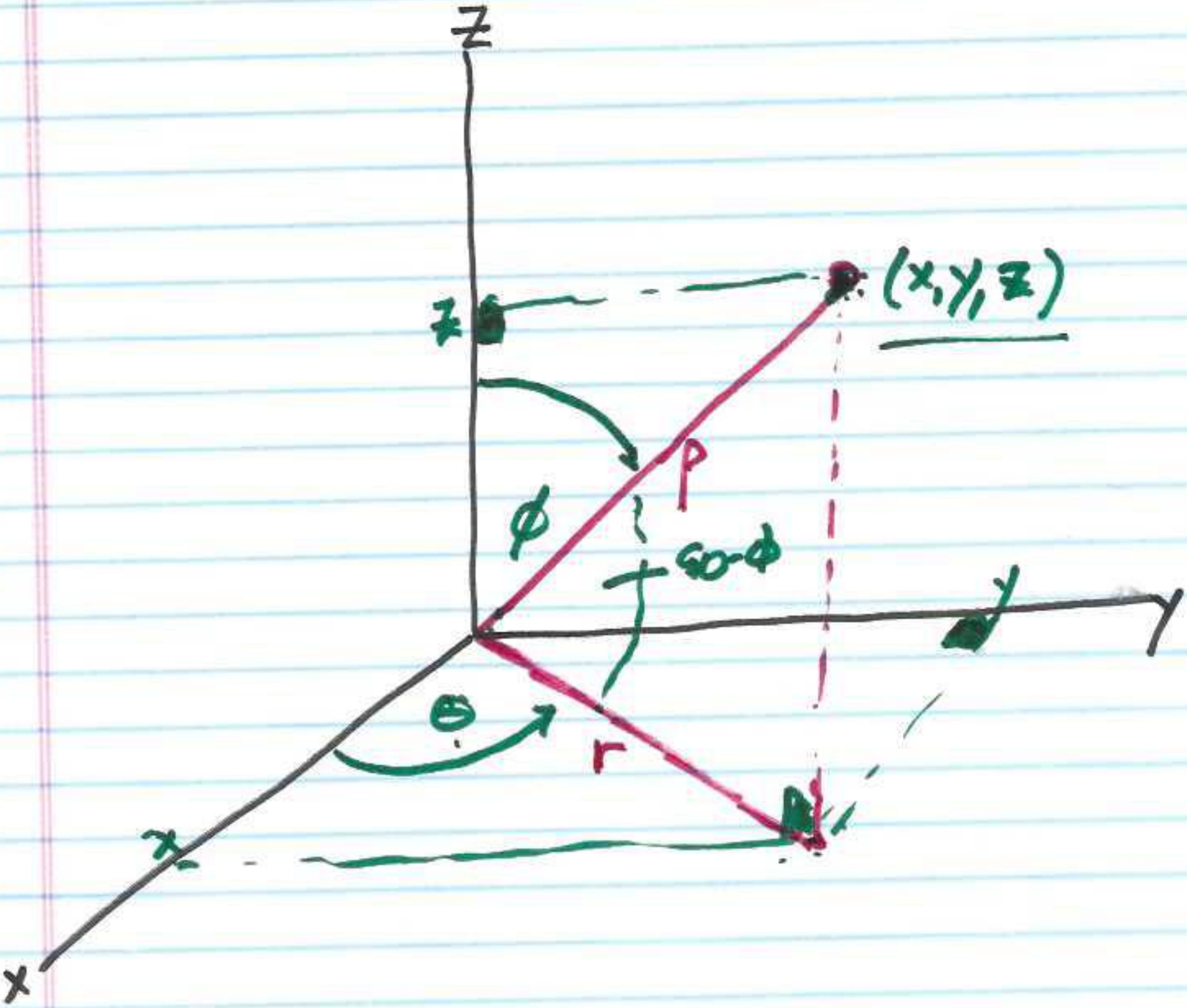
$$\iiint f(r,\theta,z)\, r\, dr\, d\theta\, dz$$

$r \neq \rho$

$$\iiint f(\rho,\theta,\phi)\, \underbrace{\quad}_{\rho^2 \sin\phi}\, d\rho\, d\theta\, d\phi$$

$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} = \underline{\rho^2 \sin\phi}$$

②



ρ = radius

θ = azmuth

ϕ = declination

3 $(x, y, z) \mapsto (\rho, \theta, \phi)$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

because: $r = \sqrt{x^2 + y^2}$ and

$$\rho = \sqrt{r^2 + z^2}.$$

$$\phi = \rho \cos(90 - \phi) = r$$

$$r = \rho \sin\phi$$

$$\frac{r}{\rho} = \sin\phi, \text{ so } \phi = \arcsin\left(\frac{r}{\rho}\right)$$

$$\arcsin \sqrt{\frac{x^2 + y^2}{x^2 + y^2 + z^2}}$$

$$\theta = \arccos\left(\frac{x}{\sqrt{x^2 + y^2}}\right) \qquad x = r\cos\theta$$

so $\theta = \arccos\left(\frac{x}{r}\right)$

$(\rho,\theta,\phi) \mapsto (x,y,z)$

$x = \overbrace{\rho \sin\phi}^{r} \cos\theta$

$y = \rho \sin\phi \sin\theta \leftarrow$

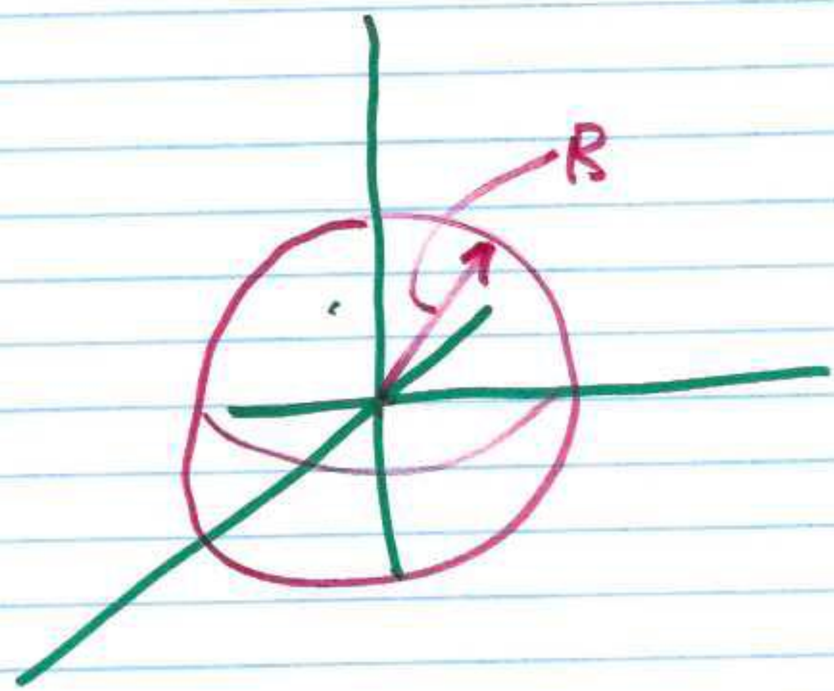
$z = \rho \cos\phi$

$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$

$$\frac{\partial y}{\partial \theta} = \rho \sin\phi \cos\theta$$

all reduces to $\rho^2 \sin\phi$

⑤



Vol of Sphere

$$V = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho^3}{3}\right]_0^R \sin\phi \, d\phi \, d\theta$$

$$\frac{R^3}{3}\int_0^{2\pi}\int_0^{\pi}\sin\phi\, d\phi\, d\theta$$

$$\frac{R^3}{3}\int_0^{2\pi}\left[-\cos\phi\right]_0^{\pi} d\theta$$

$$\frac{R^3}{3}\int_0^{2\pi}(1+1)\,d\theta = R^3\int_0^{2\pi} 2\,d\theta$$

$$= R^3\cdot\frac{2}{3}\cdot 2\pi = \boxed{\frac{4}{3}\pi R^3}$$

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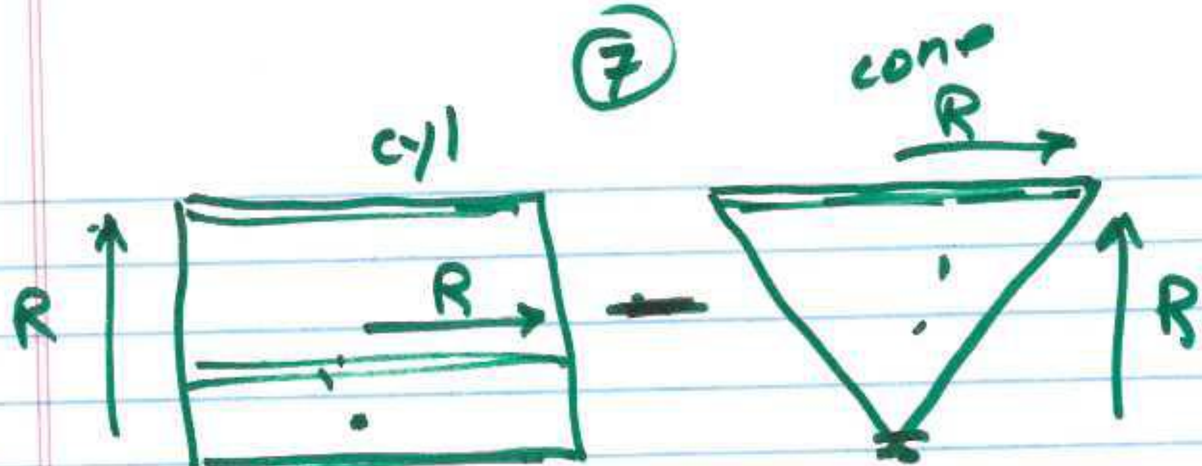
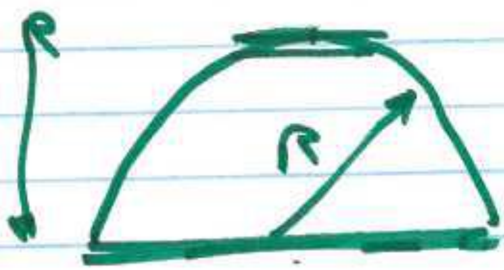
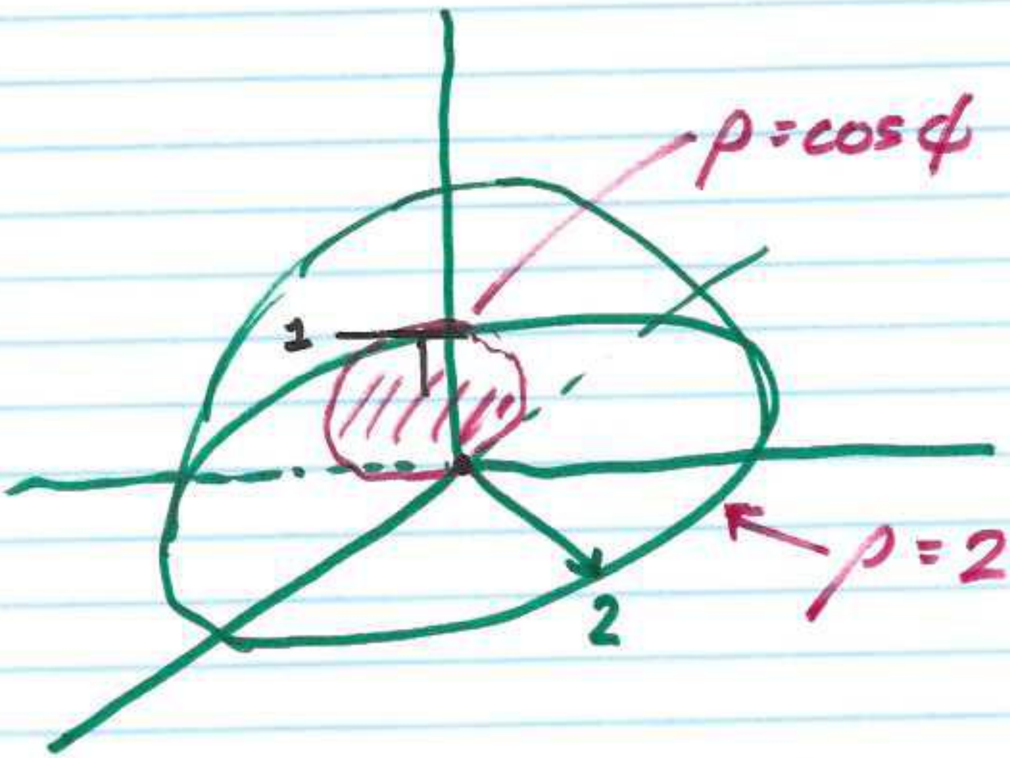




55 $\rho = \cos\phi$ sphere

$\rho = 2 \quad z \geq 0$ hemisphere



$$\frac{2}{3}\pi(8) = \frac{16}{3}\pi \text{ vol hemi}$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \leftarrow \underline{\text{sphere}}$$

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Vol interior sphere $\int_0^{2\pi}\int_0^{\pi}\int_0^{\cos\phi} \rho^2 \sin\phi \, \cancel{d\rho}\, d\phi\, d\theta$

$d\rho\, d\phi\, d\theta$

$$\int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho^3}{3}\right]_0^{\cos\phi} \sin\phi\, d\phi\, d\theta$$

$$\frac{1}{3}\int_0^{2\pi}\int_0^{\pi} \underbrace{\cos^3\phi \sin\phi}\, d\phi \, .. \, d\theta$$

$\int \cos^3\phi \sin\phi\, d\phi$ let $\cos\phi = u$

$$\int u^3(-du) = -\frac{u^4}{4}$$

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$$\text{Vol} = \frac{1}{3}\int_0^{2\pi} \underbrace{\left[\left(-\frac{1}{4}\right)\cos^4\phi\right]_0^{\pi}}_{\frac{1}{2}} d\theta$$

$$= \frac{1}{6}\int_0^{2\pi} d\theta = \frac{\pi}{3}$$

$$\frac{16}{3}\pi - \frac{\pi}{3} = 5\pi$$ final answer entire prob.