BHVAH

late 11/12 (1)

Series:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$\vdots$

$S_n = a_1 + \cdots + a_n$

$\downarrow$

$\lim_{n \to \infty} S_n$

Geometric Series: $a_0 ; r \qquad |r| < 1$

$$S = \frac{a_0}{1 - r}$$

Coefficient of Restitution:



drop height = L
return ht = αL
then COR = α

If a ball with COR .99 is dropped from 1 m. How far does it ultimately travel?

(2)

$$1 + 2{,}99 + 2\cdot(.99)^2 + 2\cdot(.99)^3 + \cdots 2\cdot(.99)^n + \cdots$$

$$1 + 2\left[\underbrace{.99 + .99^2 + .99^3 + \cdots .99^n + \cdots}\right]$$

$$\frac{.99}{1-.99} = 99$$

So total travel is $1 + 2\cdot 99 = 199$ [m].

Another series that can be explicitly summed is a so-called telescoping series.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1 \quad \Big| \quad \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} = \underline{\zeta(2)}$$

Trick: use PFD $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

reg: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$

PFD: $\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots \rightarrow$

Test for Divergence of $\sum_{n=1}^{\infty} a_n$

If $\lim_{n\to\infty} a_n \neq 0$ then divergence

Facts:

1) You can add/delete finitely many terms without convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

Prob's

8) $\sum_{n=2}^{\infty} \frac{1}{4^n}$; $\sum_{n=0}^{\infty} \frac{1}{4^n} = \underbrace{1 + \frac{1}{4}} + \frac{1}{16} + \frac{1}{64} \cdots$

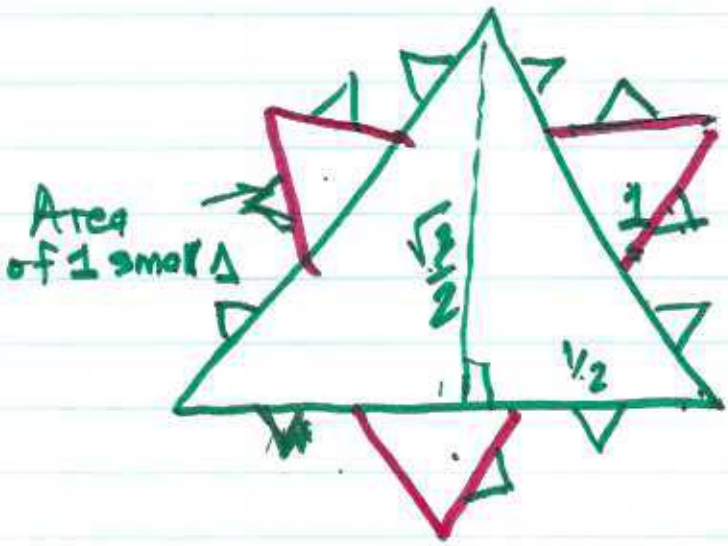
$$\frac{1}{1-\frac{1}{4}} = \frac{4}{3} - 1 - \frac{1}{4} = \boxed{\frac{1}{12}}.$$

14) $\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right) = 2\underbrace{\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n}$

$$\frac{2 \cdot 1}{1-\frac{2}{5}} = \frac{2}{\frac{3}{5}} = \boxed{\frac{10}{3}}$$

(4)

Helge von Koch



$P_1 = 3$

$A_1 = \frac{1}{2}(1)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4}$

$P_2 = 4$

$A_2 = \frac{\sqrt{3}}{4} + 3 \cdot \frac{\sqrt{3}}{36}$

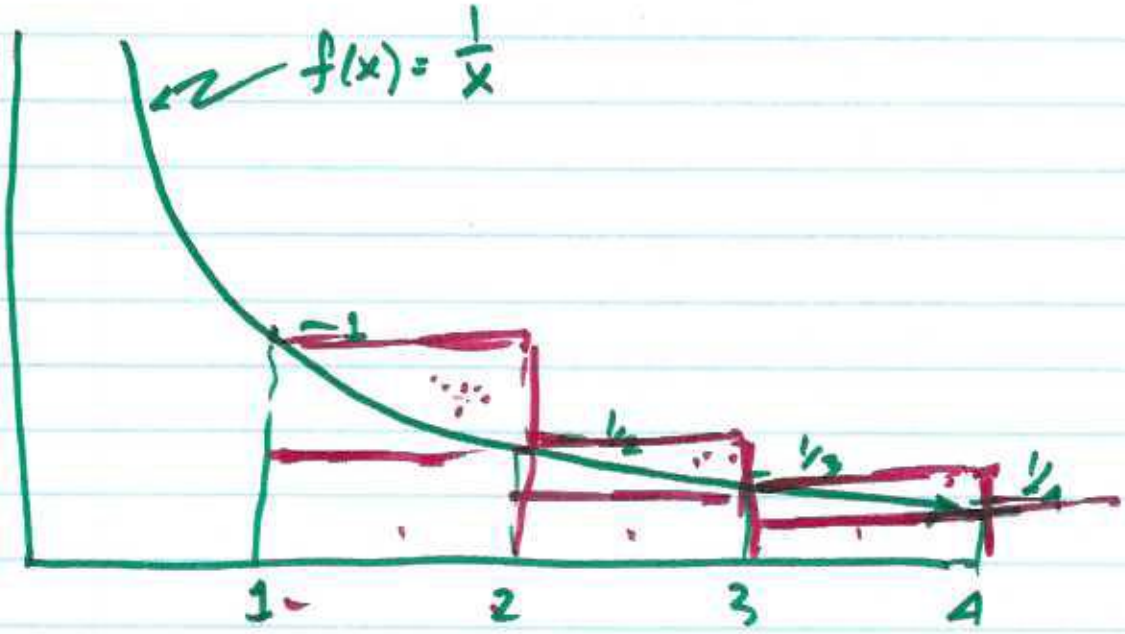
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$\lim_{n\to\infty} P_n = \infty$

$\lim_{n\to\infty} A_n = \frac{8}{5}(3)$

$A = \sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$

(5)



$$\int_1^\infty \frac{dx}{x} = \ln x \Big|_1^\infty = \infty - 0 = \infty$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \geq \int_1^\infty \frac{dx}{x} = \infty$$

Ex: $\sum_{n=1}^{\infty} \frac{n}{n^2+1} = ?$

$$\int_1^{\infty} \frac{x\,dx}{x^2+1} = ?$$ Let $u = x^2+1$

$du = 2x\,dx$

$$\int_2^{\infty} \frac{(du/2)}{u} = \frac{1}{2}\int_2^{\infty} \frac{du}{u} = \frac{1}{2}\ln u \Big|_2^{\infty} = \infty$$

Ex: $\sum_{n=1}^{\infty} n \tan\left(\frac{1}{n}\right)$

$$\rightarrow \int_1^{\infty} x \tan\frac{1}{x}\,dx$$

Let $u = \frac{1}{x} \Rightarrow \int_1^{0} \frac{1}{u}\tan u$

$du = -\frac{1}{x^2}dx$

$* dx = -du \cdot x^2$

$= -\frac{du}{u^2}$

$$\int_{\infty}^{0} \frac{1}{u} \tan u \left(-\frac{du}{u^2}\right)$$

$$= \int_{0}^{\infty} \frac{\tan u \, du}{u^3} \quad \text{– hold}$$

Check $\lim_{n\to\infty} n \tan\frac{1}{n}$

$$\frac{\tan 1/n}{1/n} = \frac{\sec^2\left(\frac{1}{n}\right)\cdot\left(-\frac{1}{n^2}\right)}{-\frac{1}{n^2}} = 1$$

So... $\sum_{n=1}^{\infty} n \tan\frac{1}{n} = \infty$

$\infty \quad 0$

$$\sum_{n=0}^{\infty} e^{-n^2} \qquad \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{2\pi}}{2}$$

$$\frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-z^2} dz = \frac{1}{2}$$



Let $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ be series such that

$$0 \le a_n \le b_n$$

1) If $\sum_{n=1}^{\infty} b_n < \infty$ then $\sum_{n=1}^{\infty} a_n < \infty$

2) If $\sum_{n=1}^{\infty} a_n = \infty$ then $\sum_{n=1}^{\infty} b_n = \infty$

Ex: Consider $\sum_{n=1}^{\infty} \frac{5}{5n-1}$. Conv or Div?

$$\frac{1}{n-\frac{1}{5}} > \frac{1}{n}$$

b_n $\quad$ a_n

Ex. $\sum_{n=0}^{\infty} \frac{1}{n!} = e$ compare w/ $\sum_{n=0}^{\infty} \frac{1}{2^n}$

$$1 + \left(1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} \cdots \frac{1}{n!}\right) \le 3$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 2$$

$$\frac{1}{2^n} > \frac{1}{n!} \qquad n! > 2^n$$

Th^{m}/ $a_n > 0$, $b_n > 0$ $\forall n \in \mathbb{N}$

(1) If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ then both series converge/diverge together.

(2) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n < \infty$, then

$$\sum a_n < \infty$$

(3) If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, $\sum b_n$ diverges, then

$$\sum_{n=4}^{\infty} a_n = \infty$$