

early
9/29

(1)

Q 7 Review:

T/F

F (1) Differential equations involve only differentials
 $dx, \frac{dy}{dx}$

T (2) Separable equations are a type of differential equation

T (3) Solutions of separable equations often involve exponentials

T (4) $\frac{dy}{dx} = \cos(xy)$ not separable
 $\frac{dy}{dx} = y \cos x \leftarrow$ is separable

T (5) $\frac{e^{-x} + e^x}{2} = \cosh x$ is this ok?

T (6) $\int_1^x \frac{dx}{x} = \ln x$ $\int_0^x \frac{dx}{x} = \ln x$
yes No

$$(2) \quad (H - H_s) = (H_0 - H_s) e^{-kt}$$

T (7) Polynomial growth is faster than logarithmic growth

T (8) $P(t) = P_0 e^{rt}$ could be an equation for principal (\$) after time t

F (9) Temperature as a function of time for a heating/cooling object varies logarithmically.

F (10) Carbon-14 dating is based on growth of radioactivity

F (11) $\ln(a+b) = \ln a + \ln b$

F (12) $\ln(a^b) = \frac{\ln a}{b}$

F (13) $\cosh^2 x + 1 = \sinh^2 x$ (True: $\sinh^2 x + 1 = \cosh^2 x$)

T (14) $-\coth x$ = $\frac{e^x + e^{-x}}{e^{-x} - e^x}$

$$\frac{\frac{e^x + e^{-x}}{2}}{\frac{e^x - e^{-x}}{2}} = \coth x$$

③

F (15) There are eight hyperbolic functions

T (16) $\operatorname{sech} x = \frac{1}{\cosh x}$ $\operatorname{sech} x \cdot \cosh x = 1$

F (17) $Q(t) = Q(0)e^{kt}$ where $k > 0$

could be a model for radioactive decay

T (18) If $L(x)$ is light intensity at surface of the ocean and $L'(x) = k \frac{L(x)}{x}$, with $k < 0$, would you say light intensity declines exponentially?

F (19) $\frac{dy}{dx} = kx^2$ represents exponential growth/decay

F (20) $a^x = \ln a \cdot e^x$ | True: $a = e^{\ln a}$
 $a^x = e^{x \cdot \ln a}$ ✓

(21) If $.5 = e^{-kt}$, t is "half-life" of some decaying quantity

(4)

F (22) 2^x grows at same rate as e^x as $x \rightarrow \infty$

$$2^x = o(e^x) \quad \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = M \quad \lim_{x \rightarrow \infty} \left(\frac{e^x}{2^x} \right) = \left(\frac{2.71}{2} \right)^x > 1$$

$$\lim_{x \rightarrow \infty} \frac{3^x}{2^x} = \left(\frac{3}{2} \right)^x = \infty$$

Also $\frac{f(x)}{g(x)} \rightarrow 0$ as $x \rightarrow \infty$ we say
 $\downarrow$ "little oh"
 $f(x) = o(g(x))$

So if $\frac{f(x)}{g(x)} \rightarrow M$ as $x \rightarrow \infty$, we say

$$f(x) = O(g(x))$$

$$[n \times n] \quad \begin{matrix} \nearrow n! \\ \circlearrowleft n^2 \end{matrix}$$

(5)

NO
SOME (23) Does $\log_3 x$ grow faster than $\ln x$ as $x \rightarrow \infty$

$$\lim_{x \rightarrow \infty} \left(\frac{\log_3 x}{\ln x} \right) \stackrel{?}{=} \lim_{x \rightarrow \infty} \left(\frac{\frac{1}{\ln 3} \cdot \frac{1}{x}}{1/x} \right)$$

$$= \frac{1}{\ln 3} > 1$$

T (24) 2^x grows faster than x^n for any n

$$\text{Look @ } \lim_{x \rightarrow \infty} \left(\frac{2^x}{x^n} \right) = \frac{e^{\ln 2 \cdot x}}{n x^{n-1}} \rightarrow$$

$$\lim_{x \rightarrow \infty} \left(\frac{(\ln 2)^n \cdot 2^x}{n!} \right) = \infty$$

T (25) Is $x + e^x = O(e^x)$ yes

$$\lim_{x \rightarrow \infty} \left(\frac{x + e^x}{e^x} \right) = 1$$

Problems ^⑥

- 1) If the carbon-14 in an object is 75% of original, estimate its age.

$$\rightarrow Q(t) = Q(0)e^{-.00012t}$$

$$\frac{Q(t)}{Q(0)} = e^{-.00012t} = .75$$

$$-.00012t = \ln(.75)$$

$$t_{1/2} \approx \underline{5776 \text{ yrs}}$$

- 2) How long for money to double

@ 10% cont. comp interest

$$P(t) = P(0)e^{rt}$$

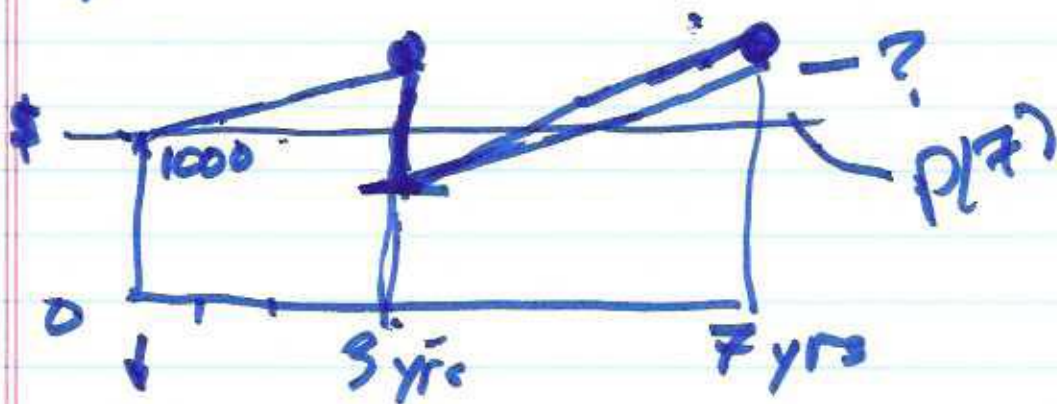
$$\underline{2 = 1e^{.1t}}$$

$$\ln 2 = .1t$$
$$.69315$$

$$t = \underline{6.9 \text{ yrs}}$$

(7)

How much money would you have if you invested \$1000 today @ 5% cont. interest and after 3 yrs you withdrew half of your account and then wait another 4 yrs.



$$P(3) = 1000e^{.05(3)} = 1000e^{.15} = \underline{\quad}$$

$$\frac{1}{2}P(3) \text{ out } \quad \frac{P(3)}{2}e^{.05(4)} :$$

$$\frac{P(3)}{2}e^{.2} = \underline{\frac{P(7)}{2}}$$

8

Logarithmic integral

$$\int \frac{\ln(\ln x) dx}{x \ln x} = ?$$

$$[\ln(\ln x)]' = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\text{Let } u = \ln(\ln x) \quad \frac{dx}{x \ln x} = \textcircled{du}$$

$$\int u du = \frac{u^2}{2} + C$$

$$\underline{\underline{\frac{[\ln(\ln x)]^2}{2} + C}}$$

⑨

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx \rightsquigarrow -\int e^u du = -e^u + C$$

$$\text{Let } u = \frac{1}{x} \text{ then } du = -\frac{dx}{x^2}$$

Newton's Law

$$H - H_s = (H_0 - H_s) e^{-kt}$$

$$\text{\$1000 grows to } 1000e^{.05(3)} = \frac{1000e^{.15}}{1.161^{.08}}$$

$$\text{remove } \frac{\$1161^{.08}}{2} \text{ which leaves } \underline{580.92}$$

$$P(7) = 580.92e^{.2} = \underline{\underline{\$709.50}}$$

$$\int \frac{\ln(\ln x) dx}{x \cdot \ln x} \leftarrow \int u du = \frac{u^2}{2} + C$$

$$[\ln(\ln x)]' = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$u = \ln(\ln x)$$

$$du = \frac{dx}{x \cdot \ln x}$$

$$\frac{[\ln(\ln x)]^2}{2} + C$$

$$e^{\ln y = f(x)}$$

$$y = e^{f(x)}$$

$$\int \frac{dy}{y} = \int f(x) dx$$

$$e^{\ln y = F(x)}$$

$$y = e^{F(x)}$$

$$\frac{dL(x)}{dx} = kL(x)$$

$$\int \frac{d(L(x))}{L(x)} = \int k dx$$

$$\ln L(x) = kx + c$$

$$L(x) = L_0 e^{kx}$$

$e^{kx} \cdot e^c$
 $\downarrow$
 L_0

Ref Sheet - $8\frac{1}{2}'' \times 11''$ both sides

Use for extended response

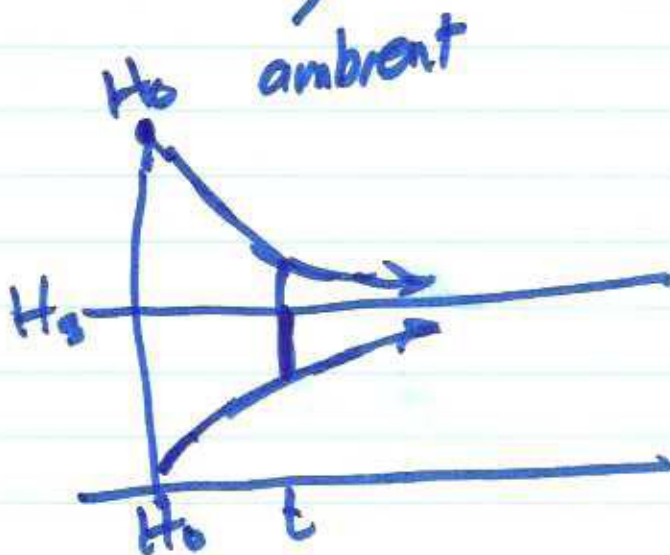
Prob types - Finance
Carbon 14
Cooling

Formulas to have: ① $P(t) = P(0)e^{rt}$ fraction
↓
 r

② $\frac{Q(t)}{Q(0)} = e^{-0.00012t}$

③ Cooling: $(H(t) - H_s) = (H(t_0) - H_s)e^{-kt}$

↑
ambient



$$\int \frac{(e^{1/x})}{x^2} dx$$

$$[e^{f(x)}]' = e^{f(x)} \cdot f'(x)$$

$$(e^{1/x})' = e^{1/x} \cdot (-\frac{1}{x^2})$$

$$\text{Let } u = \frac{1}{x}$$

$$-\frac{dx}{x^2} = du$$

$$-\int e^u du = -e^u + C$$

$$\downarrow$$
$$-e^{1/x} + C$$

$$\lim_{x \rightarrow \infty} \frac{3^x}{2^x} = ? \quad \left(\frac{3}{2}\right)^x \nearrow \infty \quad \left(\frac{2}{3}\right)^x \rightarrow 0$$

Ques x^n or 2^x for any n $2^x = (e^{\ln 2})^x$

$$\lim_{x \rightarrow \infty} \left(\frac{2^x}{x^n} \right) = \frac{\ln 2 \cdot 2^x}{n x^{n-1}} \quad \frac{\infty}{\infty}$$

$$\frac{(\ln 2)^2 \cdot 2^x}{n(n-1) x^{n-2}} \dots \frac{(\ln 2)^n \cdot 2^x}{n!}$$

$$= \infty$$

Compare $\ln x$ w/ x^2

$$\lim_{x \rightarrow \infty} \left(\frac{x^2}{\ln x} \right) = \lim_{x \rightarrow \infty} \left(\frac{2x}{\frac{1}{x}} \right) = \left(\frac{2x^2}{1} \right) \quad \downarrow$$