LC36A

early 11-12 ①

A few problems from Ch 10.1

div #37) Does $a_n = \frac{n^2-2n+1}{n-1}$ converge or div?

$a_n = n-1 \to \infty$ as $n \to \infty$

#64) $a_n = \ln n - \ln(n+1)$

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aside
- $\infty \cdot 0 = 0$
- $\infty \cdot \infty = \infty$
- $\infty + \infty = \infty$
- $\infty - \infty$ = not defined
- $\frac{\infty}{\infty}$ = not defined

$$\lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right) = \ln\left[\lim_{n\to\infty}\left(\frac{n}{n+1}\right)\right] = 0$$

#85) $a_n = \tanh n = \frac{e^n - e^{-n}}{e^n + e^{-n}}$

$\div e^n \Rightarrow a_n = \frac{1-e^{-2n}}{1+e^{-2n}}$ converges to 1

#87) $\frac{n^2}{2n-1} \sin\left(\frac{1}{n}\right)$

$\downarrow$

0

$\frac{1}{\frac{2}{n} - \frac{1}{n^2}}$ $\qquad \infty \cdot 0 = 0$

$\frac{1}{0} \rightsquigarrow \infty$

10.2 Series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots >$$

Sequence of partial sums:

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$\vdots$

$S_n = a_1 + \cdots + a_n = \sum_{k=1}^{n}$

$$\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} S_n$$

(3)

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - + \ldots \frac{x^{2n+1}}{(2n+1)!} + \cdots$$

$$\int \sin x\,dx = \int x\,dx - \int \frac{x^3}{6}\,dx + - \int \ldots$$

$$\int e^{-z^2}\,dz$$

std normal distribution

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

Let $x = -z^2$

$$\int e^{-z^2}\,dz = \int 1 + \int -z^2 + \int \frac{z^4}{2!} \int \frac{z^6}{3!} + - - \Big| \ldots$$

Def^{n} An infinite series converges to L if the sequence of partial sums converges to L.

Geometric series $\sum_{n=0}^{\infty} a_n$

a_0

$a_1 = ra_0$ $\quad |r| < 1$

$a_2 = ra_1 = r^2 a_0$

$a_3 = r^3 a_0$

$\vdots$

$S = a_0 + a_0 r + a_0 r^2 + a_0 r^3 + a_0 r^4 + \cdots$

$rS = ra_0 + a_0 r^2 + a_0 r^3 + a_0 r^4 + a_0 r^5 + \cdots$

$S - rS = S(1-r) = a_0 \Rightarrow \boxed{S = \frac{a_0}{1-r}}$

(5)

$$\sum_{n=0}^{\infty} 2^{-n} = ? \quad \frac{1}{1-(\frac{1}{2})} = 2$$

$$2^0 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + \cdots 2^{-n} + \cdots$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

$a_0 = 1 \quad r = \frac{1}{2}$

Ex $a_0 = 1/9 \quad r = \frac{1}{3}$

$$\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \cdots$$

$$\sum_{n=0}^{\infty} a_n = \frac{1/9}{1-1/3} = \frac{1/9}{2/3} = \frac{1}{9} \cdot \frac{3}{2} = \frac{1}{6}$$

Telescoping Series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = ?$$

(6)

Write $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ ✓

$a_1 = \frac{1}{1\cdot 2} = 1 - \frac{1}{2}$

$a_2 = \frac{1}{2\cdot 3} = \frac{1}{2} - \frac{1}{3}$

$a_3 = \frac{1}{3\cdot 4} = \frac{1}{3} - \frac{1}{4}$

$\vdots \quad \vdots \quad ;$

$\sum_{n=1}^{\infty} a_n = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) -$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$

Facts:

1) If $\sum_{n=1}^{\infty} a_n < \infty$ then $\lim_{n\to\infty} a_n = 0$

2) Counterexample $\sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series)

3) If $\sum_{n=1}^{\infty} a_n < \infty$, then $\sum_{n=1}^{\infty} k a_n < \infty$ $k \in \mathbb{R}$

4) If $\sum_{n=1}^{\infty} a_n < \infty$ but $\sum_{n=1}^{\infty} b_n = \infty$

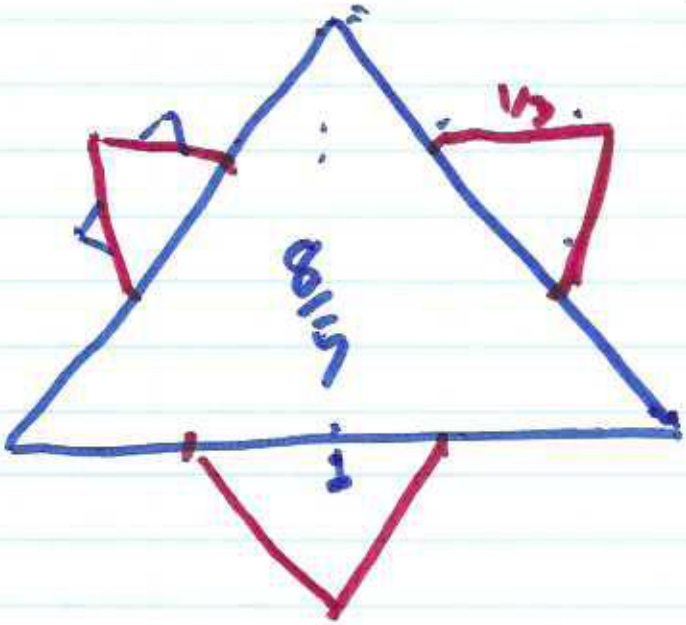
What about $\sum_{n=1}^{\infty} (a_n - b_n) = ?$

5) You can add/delete finitely many terms of any series without affecting convergence/divergence.

#19) $\sum_{n=0}^{\infty}\left(\frac{2}{e}\right)^n = ?$ $\quad \frac{1}{1-\frac{2}{e}} = \left(\frac{e}{e-2}\right)$

#14) $\sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right) = 2\sum_{n=0}^{\infty}\frac{2^n}{5^n} = 2\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n$

$$2\left[1+\frac{2}{5}+\frac{4}{25}\cdots\right] = 2\left(\frac{1}{1-2/5}\right) = 10/3$$



von Koch's snowflake

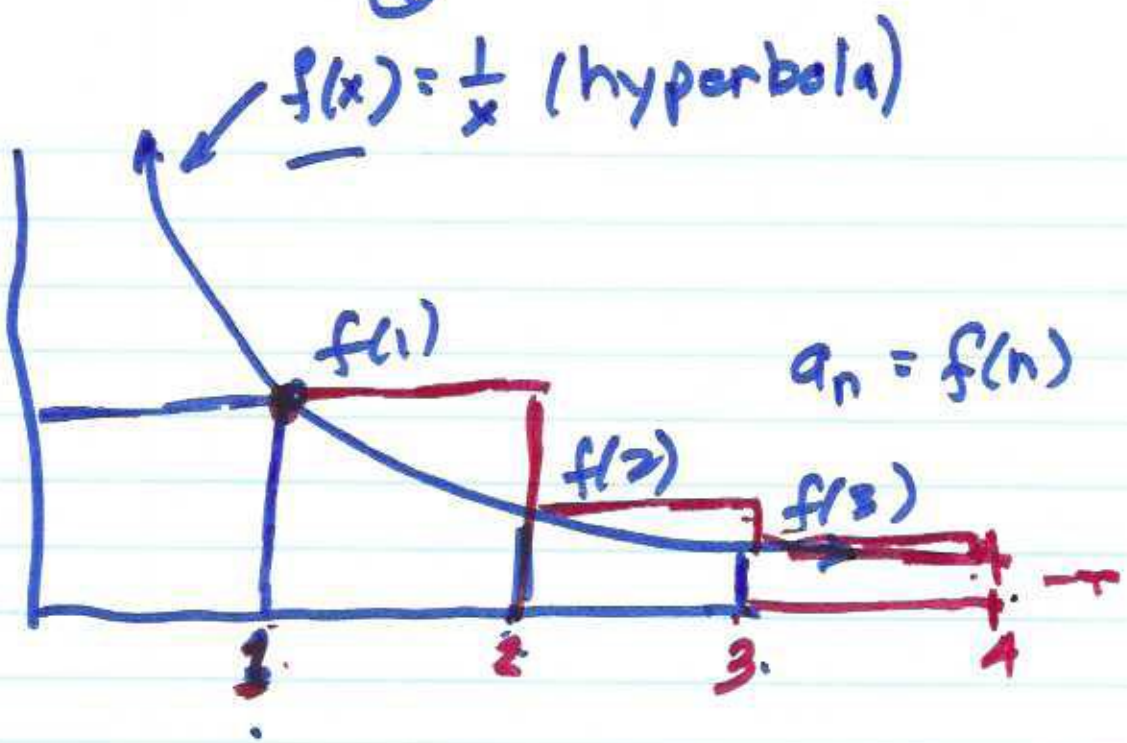
$A_1 = \frac{\sqrt{3}}{4}$

$P_1 = 3$

A_2

$\lim_{n\to\infty} P_n$

④



$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots = \infty$$

$$\int_1^{\infty} \frac{dx}{x} = \ln x \Big|_1^{\infty} = \ln(\infty) - 0 = \ln \infty_{\infty}$$

Conditions stated imply

$$\int_1^{\infty} f(x)\,dx \text{ converges/diverges in tandem } \sum_{n=1}^{\infty} a_n$$

(10)

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} \cdots$

$1 \geq \frac{1}{2}$ $\frac{1}{3} \geq \frac{1}{2}$ $\geq \frac{1}{2}$ $\geq \frac{1}{2}$ $\geq \frac{1}{2}$

Look at $\sum_{n=1}^{\infty} \frac{1}{(n^2+1)}$ $\frac{1}{x^2+1}$

$\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \cdots$

$\int_1^{\infty} \frac{dx}{x^2+1} = \arctan x \Big|_1^{\infty} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ Consider $\int_1^{\infty} \frac{dx}{x^2} = \left[-\frac{1}{x}\right]_1^{\infty} = 1$

$\searrow \frac{\pi^2}{6}$

#36 $\sum_{n=1}^{\infty} n \tan\frac{1}{n}$ = C/D?

Consider $\int_1^{\infty} x \tan\frac{1}{x}\,dx$

$du = -\frac{1}{x^2}dx$

Let $u = \frac{1}{x}$ $\int_1^0 \frac{1}{u}\tan u\,dx$

$du = -u^2 dx$

$dx = -\frac{du}{u^2}$

$$\int_0^1 \tan u \frac{du}{u^2} = \int_0^1 \frac{\tan u\,du}{u^3}$$

$$\sum_{n=1}^{\infty} \frac{n}{n+1} = ?$$

$u = \frac{1}{x}$ $du = -\frac{dx}{x^2}$

$x = \frac{1}{u}$

$$\int_1^{\infty} \frac{x}{x+1}\,dx \to \infty$$

$$\int_1^0 \frac{\frac{1}{u}}{\frac{1}{u}+1} -u^2\,du$$