LHK96

Review for Test on:

Direct Products/Sums
Normality
Homomorphisms

Define: external direct product:

$$G_1 \oplus G_2 = \{(g,h) : g \in G_1,\ h \in G_2\}$$

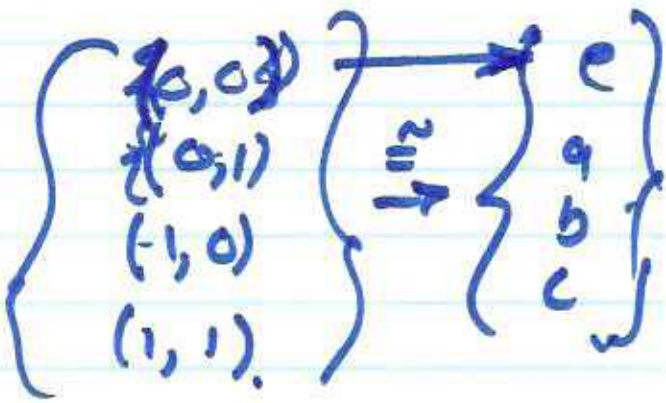
$$(g_1,h_1) * (g_2,h_2) = ?$$

$$(g_1 g_2, h_1 h_2)$$

g_1 in G_1, h_1 in G_2

$\cong$ isomy

Ex. $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \cong V_4$



(2)

Should know how to find the order of an element in an external direct product ft.

$\mathbb{Z}_5 \oplus S_3$

Given $(2, \sigma_{123})$, what is its order?

Want to take powers to achieve identity in both $\mathbb{Z}_5$ & S_3.

$|2| = 5$ in $\mathbb{Z}_5$

$|\sigma_{123}| = 3$ in S_3

$\text{lcm}(3,5) = 15$, so $(2, \sigma_{123})^{15} = (0, (1))$

$|\mathbb{Z}_3 \oplus \mathbb{Z}_6 \oplus \mathbb{Z}_9|$

$\downarrow$ $\downarrow$

3

$\text{lcm}(3,6,9) = \underline{18}$

What is criterion for $G_1 \oplus G_2$ to be cyclic ?

$\gcd(|G_1|, |G_2|) = 1$

Is $\mathbb{Z}_3 \oplus \mathbb{Z}_5$ cyclic (relatively prime)

how about $\mathbb{Z}_3 \oplus \mathbb{Z}_6$ Not cyclic (not relatively prime)

$H \triangleleft G$

$ghg^{-1} \in H$ → $\forall$ H normal

not $\forall$ H not normal

G

|

H

local condition $ghg^{-1} = h' \in H$

global condition $ghg^{-1} \in H$

$gHg^{-1} \subseteq H$

$A \subseteq B$; $B \subseteq A \Rightarrow A = B$

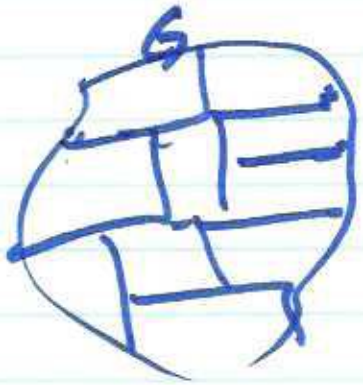
Factor Group

If $N \triangleleft G$ then construct G/N =

set of all cosets of N in G.

$(g_1 N)(g_2 N) \stackrel{\text{def}}{=} (g_1 g_2 N)$

$|G/N| = \frac{|G|}{|N|}$



G/Z Thm

Z Zentrum

Given arbitrary group G if $G/Z \cong Z_7$

$G/Z \cong V_4$ $G/Z \cong Z_6$

$G \geq H_1, H_2$ $\quad G \cong H_1 \oplus H_2$

$H_1 \cap H_2 = \{e_G\}$

$H_1 \triangleleft G, H_2 \triangleleft G$

$G = H_1 H_2 = \{(h_1, h_2) : h_i \in H_i\}$

Define homomorphism :

epimorphism	surjective homomorphism
monomorphism	injective "
isomorphism	has both
endomorphism	back to itself
automorphism	

kernel: $\phi : G \to H$

$\ker\phi = \{g \in G : \phi(g) = e_H\}$

$\phi(e_G e_G) = \phi(e_G)\phi(e_G)$

$\phi(e_G) = e_H$

What if $\ker\phi = \{e_G\}$

Is ϕ injective

Is $\ker\phi \leq G$! Yes but why?

Given $\phi : G \to H$ is

If $K \leq G$

Is $\phi(K) \leq H$

(7)

Given $\phi : G \to \bar{G}$

$\frac{G}{\ker\phi} \cong \operatorname{Im}\phi \leq \bar{G}$

→ show normal

